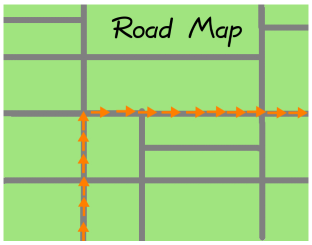
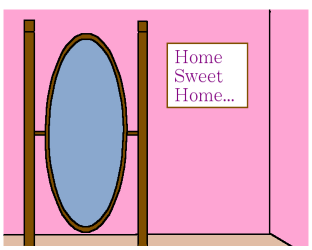
Module 0.2: Diagnostic One: Calculator Skills



As stated in the introduction to Chapter 0, this book opens with two diagnostics. The first diagnostic, presented below, mainly consists of a sequence of calculations that you are asked to perform, according to the rules of some formula. Each of the formulas below is one that we will use elsewhere in this book, and comes from some economic or financial application. It is likely that you have never seen most of these formulas before. That is not the point.

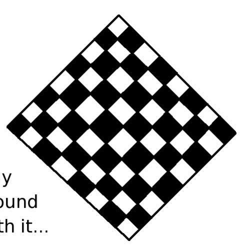
If you are presented with a formula (regardless if you've seen it before or not) and with all the data that the formula requires, either you will get a correct answer, or you will get an incorrect answer. The causes of an incorrect answer can be an incorrect key press on your calculator, non-familiarity of exactly how your calculator works, or a misunderstanding of the order of operations. It is the goal of this diagnostic to help you identify any calculator or order-of-operations misunderstandings that you might have.

After you have completed all the problems, all the answers are given on Page 57. Furthermore, all the problems (and a few more as well) are solved in detail, in the next module.



A Pause for Reflection...
I have four quick thoughts for you before we begin the diagnostic:

- Of course, since part of this diagnostic is meant to measure your ability to use your calculator, it is *very* important that you use the *specific* calculator that you will be using during exams. Different professors will place different restrictions, and so you really should check with your instructor to find out what the rules are, and which calculators are compliant.
- As I mentioned on Page 12, this book uses six significant figures as its normal standard of accuracy. However, it might be better to just go ahead and write all the digits that your calculator presents if you like, or to stop at six significant digits if you prefer.
- The diagnostic is intended to be completed in one hour—however, in the past students have taken as little as 25 minutes and as much as 90 minutes to complete the diagnostic. Do not focus on speed, but instead focus on accuracy.



0-2-1

1: Before we start practicing with formulas, we're going to work very briefly with negative numbers. As it turns out, negative numbers come up a lot in economics and finance, but they don't necessarily come up in everyday life very often. Therefore, sometimes students forget the rules of negative numbers.

Here are some practice questions.

(a.) Compute $(-4) \times (-5) + 2$.

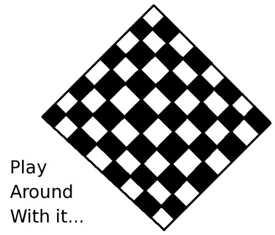
(b.) Compute $2 \times (-3) + (-8)$.

(c.) Compute $5 + (-3) \times (-7)$.

(d.) Compute $(-11) - (-9) + (-8)$.

(e.) Compute $(-4) \times (-3) \times (-2)$.

(f.) Compute $-6 + (-3) \times (11)$.



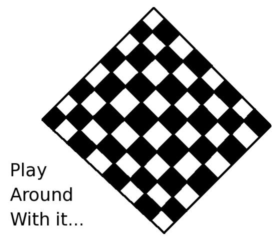
0-2-2

2: The following formula governs simple interest

$$A = P(1 + rt)$$

where A is the amount at the end, P is the principal (the amount at the beginning), t is the number of years, and r is the annual interest rate.

For now, don't worry if you've never seen simple interest before, as we will study that topic on Page 254. Meanwhile, evaluate this formula to find A given the values $P = 5800$, $r = 0.07$ and $t = 2$.

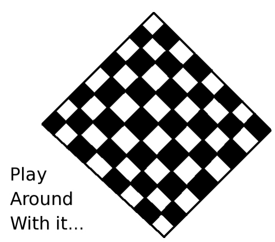


0-2-3

3: The following formula is used to find the sum S of a geometric progression, whose first member is a and whose last member z, but whose common ratio is c_r.

$$S = \frac{a - c_r z}{1 - c_r}$$

Don't worry if you've never heard of geometric progressions before, because we'll cover that in detail on Page **??**. For now, evaluate this formula to find S where $a = 4096$, $z = 1$, and $c_r = 1/4$.

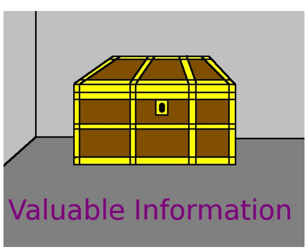


0-2-4

4: Of all formulas in the entire book, the compound interest formula is among the ones that we will use most frequently. We will study it in detail, starting on Page 278. The formula is written

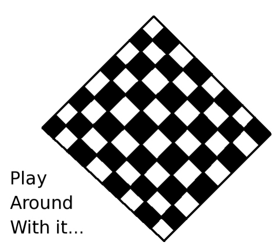
$$A = P(1 + i)^n$$

Evaluate this formula to find A where $r = 0.06$, $P = 2400$, $t = 5$, $m = 12$, $i = r/m$ and $n = mt$.



Let's take a break, momentarily, from exploring formulas. We need two common pieces of terminology.

- When we write 3.5 as 7/2, we say that we are writing an *improper fraction*—even though it is the form that mathematicians prefer.
- Likewise when we write 3.5 as $3\frac{1}{2}$, we say this is a *mixed number*, which I guess would be more properly written as $3 + \frac{1}{2}$.



0-2-5

5: With the vocabulary from the previous box in mind, perform the following conversions.

(a.) Write 72/5 as a mixed number.

(b.) Write $3\frac{2}{11}$ as an improper fraction.

(c.) Write $5\frac{7}{13}$ as an improper fraction.

(d.) Write 18/7 as a mixed number.

Play Around With it...

0-2-6

6: The next formula is used when saving for retirement, or saving up for anything else. It is called the future value formula, and we will study it in detail on Page 641.

$$FV = c \cdot \frac{(1+i)^n - 1}{i}$$

Evaluate this formula to find FV, given that $c = 165$, $n = 52 \times 10$, $r = 0.07$, and $i = r/52$.

Play Around With it...

0-2-7

7: The following formula, which can be traced back to Bernoulli, will tell us the probability p that two devices are defective among a shipment of n devices, if q is the underlying probability that any particular individual device is defective.

$$p = \frac{n(n-1)}{2} \cdot q^2(1-q)^{n-2}$$

Most students will have never seen this formula before, but if you have, then I'm very impressed; we will learn about it on Page 1014. Evaluate this formula to find p using $n = 28$, $q = 0.02$.

Play Around With it...

0-2-8

8: Probably the most famous named formula in all of mathematics is the quadratic formula, written below.

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We'll look at this formula a lot, starting at Page 103. For now, I'd like you to use this formula to solve

$$x^2 - 68x + 1147 = 0$$

and you will discover two solutions. I would like the larger one, please.

Play Around With it...

0-2-9

9: Here is a formula that governs mortgages and car loans.

$$PV = c \cdot \frac{1 - (1+i)^{-n}}{i}$$

where c is the monthly payment, PV is the present value of the loan (how much you can borrow), and n is the number of payments, while i is the interest rate. We will study this formula starting on Page 579, but at this time, use this formula to compute PV given that $n = 360$, $c = 830$, $r = 0.07$, and $i = r/12$.

Play
Around
With it...

\# 0-2-10

\# 10: When designing a soda can, an oil drum, or any cylinder, the minimal amount of metal should be used to keep costs down, which means minimizing the surface area. For any volume V, that minimum will be achieved by radius r and height h where

$$r = \sqrt[3]{\frac{V}{2\pi}} \text{ and } h = \sqrt[3]{\frac{4V}{\pi}}$$

remembering that $\pi \approx 3.14159265358979323846 26 \cdots$.

In all likelihood this formula looks very strange but we'll analyze it further on Page 1169. At this time, tell me what the height of an oil drum should be if it is to have a volume of 13 cubic feet, but use the minimum amount of metal (thus having the minimum surface area.)

Okay, we're done exploring formulas now. We just have a bit more to practice—a few quick problems—and then the diagnostic will be complete.

Play
Around
With it...

\# 0-2-11

\# 11: Please write the following numbers in order, sorted from smallest to largest:

$$\frac{1}{\sqrt{5}},\;\; \pi/6,\;\; 5/17,\;\; 1,\;\; 0.61,\;\; 6/5,\;\; (7/8)^3$$

The next four boxes are going to look like extremely easy questions, perhaps to the point of being insulting. Please don't be insulted—the purpose of these questions is to illuminate an extremely common misconception among students. This misconception has to do with parentheses, and many students will "fall into this pit" on quizzes and exams. The purpose of these boxes is to shine the light on that pit, so that you can see it and avoid falling into it.

Play
Around
With it...

\# 0-2-12

Question # 12:

(a.) What is $17^2 + 4^2$?

(b.) What is $(17 + 4)^2$?

(c.) Are your answers to Part (a) and Part (b) of this question the same? (Yes or No.)

Play
Around
With it...

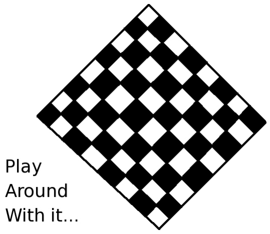
\# 0-2-13

Question # 13:

(a.) What is $5^2 + 7^2$?

(b.) What is $(5 + 7)^2$?

(c.) Are your answers to Part (a) and Part (b) of this question the same? (Yes or No.)



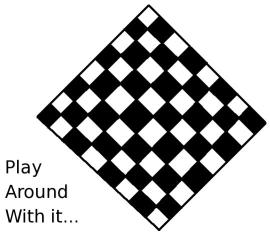
\# 0-2-14

Question # 14:

(a.) What is $3^3 + 4^3$?

(b.) What is $(3+4)^3$?

(c.) Are your answers to Part (a) and Part (b) of this question the same? (Yes or No.)



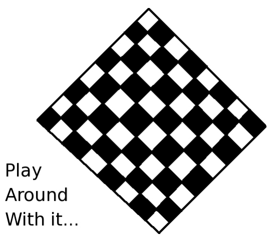
\# 0-2-15

Question # 15:

(a.) What is $\sqrt{15^2 + 12^2}$?

(b.) What is $15 + 12$?

(c.) Are your answers to Part (a) and Part (b) of this question the same? (Yes or No.)



\# 0-2-16

Question # 16: If one must go 105 miles at 80 mph then that would require 1 hour, 18 minutes, and 45 seconds. In a similar manner, fill in all the spots below, to construct a mathematically correct sentence in this form.

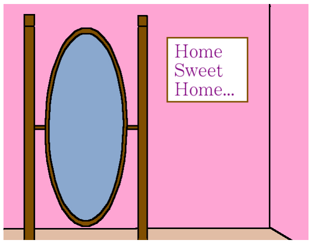
(a.) If one must go 105 miles at 75 mph then that would likewise require _______ hour(s), _______ minutes, and _______ seconds.

(b.) If one must go 105 miles at 70 mph then that would likewise require _______ hour(s), _______ minutes, and _______ seconds.

(c.) If one must go 105 miles at 63 mph then that would likewise require _______ hour(s), _______ minutes, and _______ seconds.

(d.) If one must go 105 miles at 60 mph then that would likewise require _______ hour(s), _______ minutes, and _______ seconds.

The first diagnostic is now complete. The answers are to be found on Page 57. However, I'd first like to discuss something very serious with you in the next box, which is on the next page.



A Pause for Reflection...
In Question 16, you saw that taking a trip of 105 miles at the speed of 70 mph takes 1 hour and 30 minutes, while at the speed of 75 mph takes 1 hour and 24 minutes. Let's think about this now.

In rural Wisconsin, where the speed limit is 65 mph, most people drive around 69–72 mph. However, if one drives 75 mph, then being 10 mph over the limit exposes one to speeding tickets. Furthermore, you have to pass other drivers often and each time you do that, there is some probability—relatively small but not zero—of a collision. In return for risking a speeding ticket and an accident, what do you gain? You gain only 6 minutes, even though you are taking a relatively long drive. Is six minutes significant in a drive of 90 minutes duration?!

Likewise, when I grew up in urban New Jersey, the speed limit was 55 mph. Most people therefore drove 59–62 mph. If you have a journey of 105 miles at 60 mph, it will take you 1 hour and 45 minutes. What happens if you risk a ticket or an accident by driving 63 mph? You save only 5 minutes, nothing more.

Let's take a moment to discuss what Questions 12c, 13c, 14c, and 15c are all about.

(a.) In general, it is not the case that $(x+y)^2 = x^2 + y^2$.

(b.) In general, it is not the case that $(x+y)^3 = x^3 + y^3$.

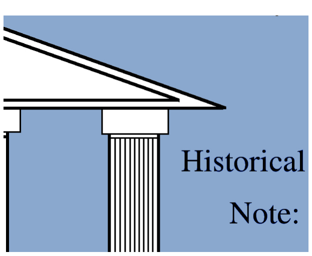
(c.) In general, it is not the case that $\sqrt{x^2+y^2} = x + y$.

For some reason, students tend to imagine that the three formulas above are true. They will take steps that would be valid if those equalities were true, such as seeing $(x+5)^2$ on one line, and writing $x^2 + 5^2$ on the next line. This is very common error, but it is also very destructive. It is particularly destructive in *Business Calculus* courses.

Out of all the infinite number of possible values for x and for y, the equation

$$(x+y)^2 = x^2 + y^2 \leftarrow \qquad \text{WRONG!}$$

will only be true if one or both variables is equal to zero. In the vast, vast majority of cases, both variables are non-zero and the move is illegal, resulting in a wrong answer for the problem. Please, do not make this mistake.



At this point, you might be thinking of formulas as a pain in the rear end, but there's another side to the story. Formulas are relatively recent, as it turns out. Even basic algebraic symbols like the plus sign and the equal sign were not introduced until the Renaissance. However, the human race has been skilled at mathematics for millennia.

The oldest written records of mathematics are found among the oldest written records of any kind at all, in the oldest written language—Sumerian. Some of the absolute oldest surviving examples of writing are tallies of the various animals of livestock in herds.

However, the Sumerians (3500s BCE—2270 BCE) did not have formulas, and therefore had to display patterns by showing the reader/student a long sequence of examples. This is extremely cumbersome for even the most basic patterns, and for more complex patterns it would just baffling.

We're going to glance at two examples now, in the next two boxes.

Consider the following computations and see if you can spot a pattern:

$$\begin{array}{rcrcrcl} 3\times 4\times 5 &=& 60 &=& 64-4 &=& 4^3-4\\ 4\times 5\times 6 &=& 120 &=& 125-5 &=& 5^3-5\\ 5\times 6\times 7 &=& 210 &=& 216-6 &=& 6^3-6\\ 6\times 7\times 8 &=& 336 &=& 343-7 &=& 7^3-7\\ 7\times 8\times 9 &=& 504 &=& 512-8 &=& 8^3-8\\ 8\times 9\times 10 &=& 720 &=& 729-9 &=& 9^3-9\\ 9\times 10\times 11 &=& 990 &=& 1000-10 &=& 10^3-10 \end{array}$$

Here's another one. . . can you spot the pattern?

$$\begin{array}{rcrcrcl} 4\times 6 &=& 24 &=& 25-1 &=& 5^2-1\\ 5\times 7 &=& 35 &=& 36-1 &=& 6^2-1\\ 6\times 8 &=& 48 &=& 49-1 &=& 7^2-1\\ 7\times 9 &=& 63 &=& 64-1 &=& 8^2-1\\ 8\times 10 &=& 80 &=& 81-1 &=& 9^2-1\\ 9\times 11 &=& 99 &=& 100-1 &=& 10^2-1 \end{array}$$

The equation governing this pattern, and the pattern in the previous box, will be revealed on Page 58.

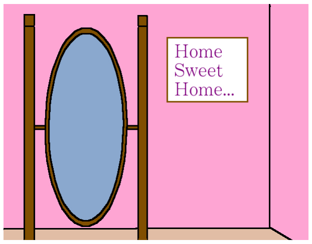
Here are the answers to the questions that form the diagnostic. The answers continue into the next box.

(1a.)	+22	(2.)	\$ 6612.00	(6.)	\$ 124,140.95
(1b.)	-14	(3.)	5461	(7.)	$0.0894189\cdots$
(1c.)	+26	(4.)	\$ 3237.24	(8.)	37
(1d.)	-10	(5a.)	$14\frac{2}{5}$	(9.)	\$ 124,755.28
(1e.)	-24	(5b.)	35/11	(10.)	$r = 1.27424\cdots$
(1f.)	-39	(5c.)	72/13		$h = 2.54849\cdots$
		(5d.)	$2\frac{4}{7}$		

Question #11: $5/17$, $1/\sqrt{5}$, $\pi/6$, 0.61, $(7/8)^3$, 1, $6/5$

(12a.)	305	(14a.)	91	(16a.)	1 hour 24 mins 0 sec
(12b.)	441	(14b.)	343	(16b.)	1 hour 30 mins 0 sec
(12c.)	No.	(14c.)	No.	(16c.)	1 hour 40 mins 0 sec
(13a.)	74	(15a.)	$19.2093\cdots$	(16d.)	1 hour 45 mins 0 sec
(13b.)	144	(15b.)	27		
(13c.)	No.	(15c.)	No.		

Since I have shown six digits of precision, then your answer should match mine in the five most significant digits. Definitely do not worry if the seventh, eighth or later digit does not match. It would be nice if the sixth significant digit matched, but it accepted scientific practice that the least significant digit shown is not as accurate as the others.



A Pause for Reflection...
Now that you've checked your answers,...

- ...if you got three or more questions wrong, then you should read the module "Order of Operations and Calculator Skills," and do all the checkerboard problems. That module will get you ready for the rest of the book. It is a very important module.
- ...if you got one or two questions wrong, then you can probably find the corresponding examples inside the module "Order of Operations and Calculator Skills," and see how to do whatever it was that you got wrong. Alternatively, you can also choose to do the entire module.
- ...if you got nothing wrong, congratulations. You should feel very confident, and after some rest, you should attempt Diagnostic Two.

Last but not least, there was the question (on Page 57) of the patterns that the Sumerians would use to express mathematical ideas, since formulas had not been invented yet.

The first pattern that you were shown represents

$$(x-1)(x)(x+1) = x^3 - x$$

and the second pattern that you were shown represents

$$(x-1)(x+1) = x^2 - 1$$

This concludes Diagnostic One. Good luck with the rest of the course.