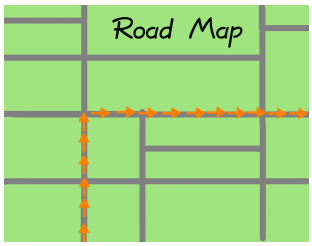
Module 0.4: Diagnostic Two: Algebra Skills



The purpose of this short 19-question diagnostic is to test precisely the specific skills from your algebra coursework that we will be using throughout this book. While these problems are a bit dry, as compared to the fun financial, scientific, and economics word problems that we'll be doing for most of the book, please take this seriously.

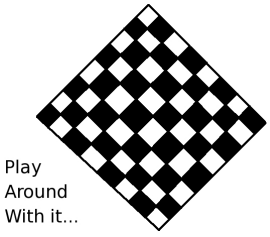
In my years of teaching, I have to come to realize that *everyone*, even the A-students, will forget a few things from their algebra coursework. The goal here is to identify precisely what you should review. All you have to do is take this diagnostic, check the answers (given on Page 92), and see which questions you have gotten correct and those which were not correct. Each question has a resource or resources associated with it, waiting for you. (For example, if you get Question #3 wrong, then you should read the module "Solving Quadratic Equations.")

This diagnostic is very different from an ordinary exam, in that we're not trying to give you some number of points. None of these questions are worth points. Each one is either something that you know how to do, or something that you need to brush up on. Each question is like an island unto itself—except that some questions are twins.

The goal is to help you verify precisely what you are expert in, and what you need to review. This way, you don't have to review anything that you already know.

Here are the guidelines for this diagnostic:

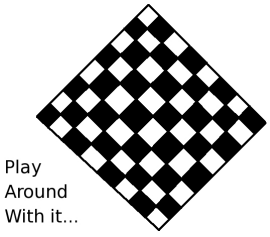
- Use whichever calculator you will be using throughout the course.
- Forecast for yourself one complete uninterrupted hour for the whole thing.
- However, do not stop until you hit two hours—some students work faster than others.
- Do not check any answers until you've finished the entire set of 19 questions.



0-4-1

Question # 1: Please solve the following equations for x.

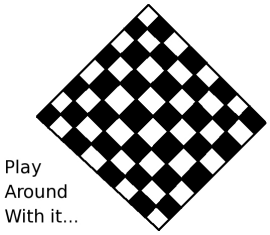
- $18x + 5 = 11x + 124$.
- $4(2x + 19) = 7x + 87$.



0-4-2

Question # 2: Convert these equations to the $y = mx + b$ format.

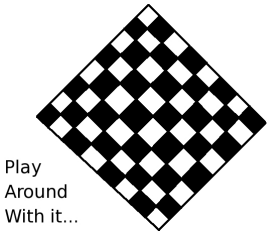
- $3x + 5y - 6 = 2x + 10y$.
- $(x/11) + (y/41) = 1$.
- $y - 12 = 4(x - 6)$.
- $\frac{3x+5y}{4} = 8x + 9y - 7$.



0-4-3

Question # 3: Please solve the following equations for x.

- $9x^2 + 5x + 3 = 5/3 - 2x$.
- $9x^2 - 100x + 400 = 2x^2 + 12x - 41$.



0-4-4

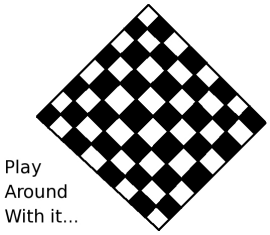
Question # 4: Please carefully graph the following lines, on the same graph.

$$5x + 4y = 3 + 4x + 3y$$

as well as

$$2x - 2y = 8 - 3y$$

over the interval $-2 \leq x \leq 10$, and the interval $-12 \leq y \leq 12$.



0-4-5

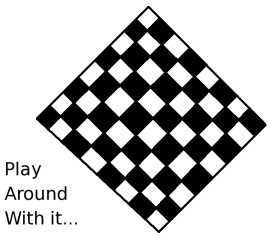
Question # 5: Please carefully graph the following lines, on the same graph.

$$7x + 4y = 10 + 2x + 2y$$

as well as

$$8x + 5y = 18 + 2x + 2y$$

over the interval $-4 \leq x \leq 6$, and the interval $-10 \leq y \leq 15$.



0-4-6

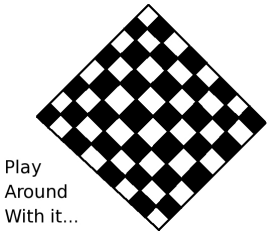
Question # 6: Solve this system of equations to find P and Q.

$$\begin{aligned} Q &= 101 - 5P \\ Q &= 3P - 27 \end{aligned}$$

[Answer: $P = 16$ and $Q = 21$.]

By the way, you probably didn't realize it, but in the previous problem you just solved a "supply & demand" problem. For example, P is the price and Q is the quantity.

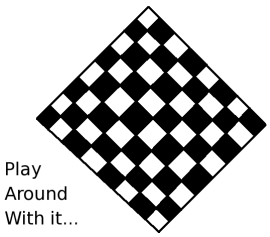
We will talk about this more in the module "Supply and Demand" on Page 435. It is really quite easy and fun—much easier than many topics in this textbook, and very revealing about the inner machinery of prices and macroeconomics.



0-4-7

Question # 7: Solve this system of equations.

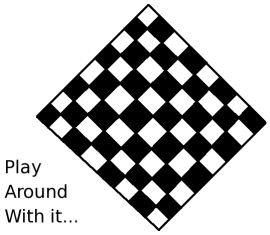
$$\begin{aligned} 8x + 5y &= 23 \\ 3x - 6y &= -15 \end{aligned}$$



0-4-8

Question # 8: Solve this system of equations.

$$\begin{aligned} 2x + 5y &= 3x - 4 + y \\ 6x + 6y &= 3x + 30 \end{aligned}$$

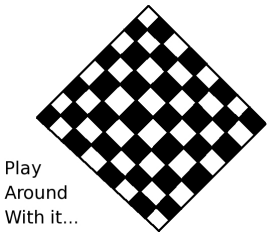


0-4-9

Question # 9: Consider the following functions:

$$f(x) = 2x + 5 \text{ as well as } g(x) = x^2 - x + 1$$

- What is $f(x) + g(x)$?
- What is $f(x) - g(x)$?
- What is $f(x) \cdot g(x)$?
- Note: that last one is a multiplication of functions.

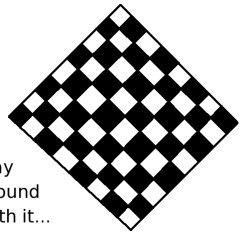


0-4-10

Question # 10: Consider the following functions:

$$a(x) = 6 + 2x \text{ as well as } b(x) = x^2 - x + 1$$

- What is $a(x) + b(x)$?
- What is $a(x) - b(x)$?
- What is $a(x) \cdot b(x)$?
- Note: that last one is a multiplication of functions.

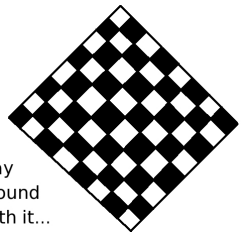


0-4-11

Question # 11: Consider the following functions:

$$f(x) = 9x - 11 \text{ as well as } g(x) = x^2 + 14x - 100$$

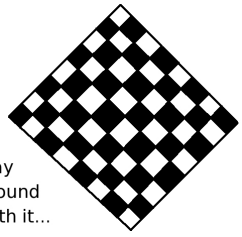
- What is $g(1)$?
- What is $f(2) + f(3)$?
- What is $f(2 + 3)$?
- Should one expect, in general, $f(2) + f(3) = f(2 + 3)$?
- What is $g(-3)$?
- Find x such that $f(x) = 1249$.
- What is $f(x) - g(x)$?
- What is $f(x) \cdot g(x)$?
- Note, that last one is a multiplication of functions.



0-4-12

Question # 12:

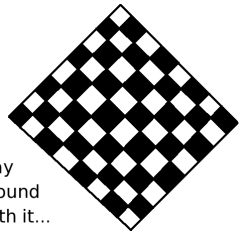
- What is $\frac{2}{7} + \frac{6}{49}$?
- What is $\frac{7}{15} - \frac{3}{10}$?
- What is $\frac{6}{11} + \frac{7}{33}$?
- What is $\frac{5}{6} - \frac{3}{10}$?
- What is $\frac{4}{5} \times \frac{5}{6}$?
- Be sure to reduce your answers to lowest terms.



0-4-13

Question # 13:

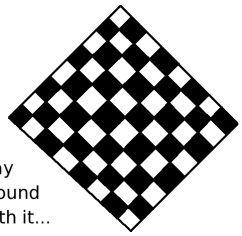
- What is $\frac{2}{5} + \frac{14}{25}$?
- What is $\frac{3}{14} - \frac{1}{21}$?
- What is $\frac{4}{11} + \frac{3}{22}$?
- What is $\frac{10}{21} - \frac{2}{35}$?
- What is $\frac{8}{21} \times \frac{7}{16}$?
- Be sure to reduce your answers to lowest terms.



0-4-14

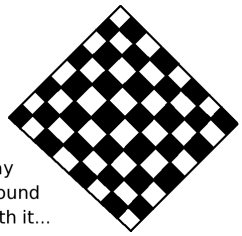
Question # 14:

- Which is bigger? 7/24 or 17/60?
- Which is bigger? 5/18 or 3/10?
- Which is bigger? 8/15 or 11/20?
- Which is bigger? 9/14 or 13/21?



0-4-15

Question # 15: A company that manufacturers toys has a new product, but the marketing department was unsure of how well the new product would sell, so they made the initial run very small. The first shipment was 80,000 toys but sold extremely well. As a result, the new shipment is 620,000 toys. The old shipment weighed 12.1 metric tons. How much will the new shipment weigh? (Clearly the weight of the shipment is directly proportional to the number of toys.)



0-4-16

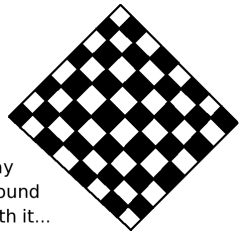
Question # 16: At a large company banquet, two corporate representatives are seated with 10 newly hired salespeople around a large round table. Five of the salespeople have each been given one of the boroughs of New York City as their territory, and the other five have each been given an entire state. They are going to be selling surgical devices to help treat a painful type of bunion.

The salespeople are curious how large their markets will be. The senior corporate representative says he has an email from the Delaware salesperson saying that there is an estimated 1562 people in Delaware with bunions severe enough to require surgery. He then tasks the junior corporate "rep" to compute the market for the ten salespeople seated around the table. The junior corporate rep does a quick internet search, and finds the following population data:

Wyoming	576,412	Staten Island	470,728
Vermont	626,011	The Bronx	1,408,473
North Dakota	699,628	Manhattan	1,619,090
Alaska	731,449	Queens	2,272,771
South Dakota	833,354	Brooklyn	2,565,635
Delaware	917,092		

This data represents July 1st, 2012 estimates.

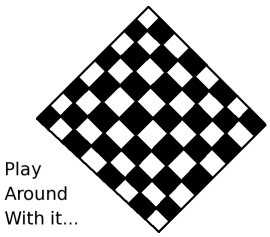
(For this problem, in order to save time, just compute the number people for South Dakota and for Brooklyn.)



0-4-17

Question #17: Convert the following numbers from scientific notation to ordinary notation.

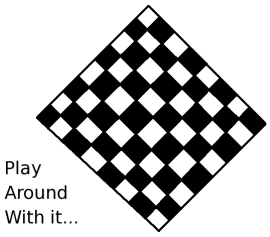
- Convert 6.392×10^{-3}.
- Convert 4.502×10^{-4}.
- Convert 7.932×10^{-2}.
- Convert 1.918×10^{-1}.



0-4-18

Question #18: Convert the following numbers from scientific notation to ordinary notation.

- Convert 8.613×10^6
- Convert 3.016×10^8
- Convert 265×10^7
- Convert 4.57801×10^4



0-4-19

Question #19: Identify the following numerals:

- What is the 2nd significant digit of 28,900,000,000?
- What is the 3rd significant digit of 0.0004236?
- What is the 4th significant digit of 352.691?
- What is the 5th significant digit of 24,879,000?
- What is the 6th significant digit of 0.006700249?

The answers for all the questions are found on the next page.
The rest of this page is left intentionally blank, so that you don't see the answers prematurely. ;-)

For Question # 1:

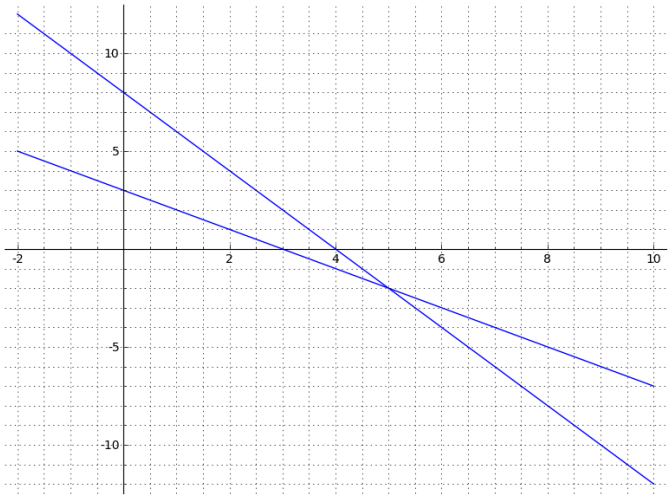
- The solution is $x = 17$.
- The solution is $x = 11$.

For Question # 2:

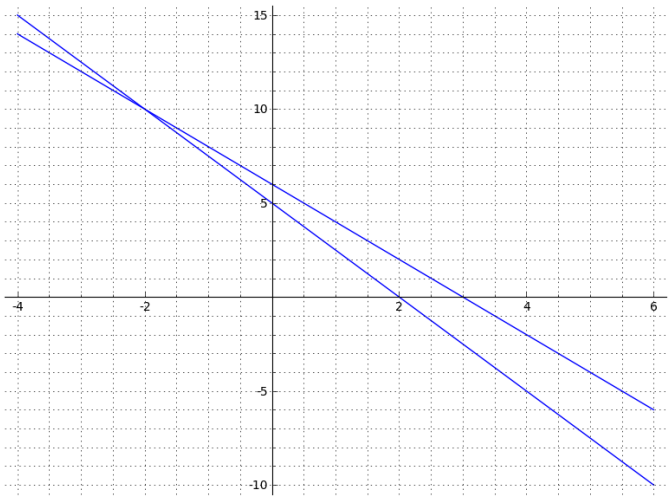
- $y = (x/5) - (6/5)$.
- $y = (-41/11)x + 41$.
- $y = 4x - 12$.
- $y = (-29/31)x + 28/31$.

For Question # 3:

- The solutions are $x = -1/3$, and $x = -4/9$.
- The solutions are $x = 9$, and $x = 7$.



Here is the graph for Question # 4. The steeper line is the second equation, and the shallower line the first equation, if you are curious.



Here is the solution to Question # 5. This time, the steeper line is the first equation, and the shallower line the second equation, if you are curious.

For Question # 6:

$$\begin{aligned} Q &= 101 - 5P \\ Q &= 3P - 27 \end{aligned}$$

Question # 7: The solution is $x = 1$ and $y = 3$.

Question # 8: The solution is $x = 8$ and $y = 1$.

Question # 9:

- $f(x) + g(x) = x^2 + x + 6$.
- $f(x) - g(x) = -x^2 + 3x + 4$.
- $f(x) \cdot g(x) = 2x^3 + 3x^2 - 3x + 5$.

Question # 10:

- $a(x) + b(x) = x^2 + x + 7$.
- $a(x) - b(x) = -x^2 + 3x + 5$.
- $a(x) \cdot b(x) = 2x^3 + 4x^2 - 4x + 6$.

Question # 11:

- What is $g(1)$? [Answer: -85.]
- What is $f(2) + f(3)$? [Answer: 23.]
- What is $f(2+3)$? [Answer: 34.]
- Should one expect, in general, $f(2) + f(3) = f(2+3)$? [Answer: Of course not!!]
- What is $g(-3)$? [Answer: -133.]
- Find x such that $f(x) = 1247$. [Answer: $x = 140$.]
- What is $f(x) - g(x)$? [Answer: $-x^2 - 5x + 89$.]
- What is $f(x) \cdot g(x)$? [Answer: $9x^3 + 115x^2 - 1054x + 1100$.]
- Note, that last one is a multiplication of functions. (Yes, it was a bit tougher than the typical problem, for sure.)

For Question # 12:

$$\begin{aligned}
\frac{2}{7} + \frac{6}{49} &= \frac{2\times 7}{49} + \frac{6}{49} = \frac{14+6}{49} = \frac{20}{49} \\
\frac{7}{15} - \frac{3}{10} &= \frac{7\times 2}{30} - \frac{3\times 3}{30} = \frac{14}{30} - \frac{9}{30} = \frac{5}{30} = \frac{1}{6} \\
\frac{6}{11} + \frac{7}{33} &= \frac{6\times 3}{33} + \frac{7}{33} = \frac{18}{33} + \frac{7}{33} = \frac{25}{33} \\
\frac{5}{6} - \frac{3}{10} &= \frac{5\times 5}{30} - \frac{3\times 3}{30} = \frac{25}{30} - \frac{9}{30} = \frac{16}{30} = \frac{8}{15} \\
\frac{4}{5} \times \frac{5}{6} &= \frac{4\times 5}{5\times 6} = \frac{4}{6} = \frac{2}{3}
\end{aligned}$$

For Question # 13:

$$\begin{aligned}
\frac{2}{5}+\frac{14}{25} &= \frac{2\times 5}{25}+\frac{14}{25}=\frac{10}{25}+\frac{14}{25}=\frac{24}{25}\\
\frac{3}{14}-\frac{1}{21} &= \frac{3\times 3}{42}-\frac{1\times 2}{42}=\frac{9}{42}-\frac{2}{42}=\frac{7}{42}=\frac{1}{6}\\
\frac{4}{11}+\frac{3}{22} &= \frac{4\times 2}{22}+\frac{3}{22}=\frac{8}{22}+\frac{3}{22}=\frac{11}{22}=\frac{1}{2}\\
\frac{10}{21}-\frac{2}{35} &= \frac{10\times 5}{105}-\frac{2\times 3}{105}=\frac{50}{105}-\frac{6}{105}=\frac{44}{105}\\
\frac{8}{21}\times\frac{7}{16} &= \frac{8\times 7}{21\times 16}=\frac{1\times 1}{3\times 2}=\frac{1}{6}
\end{aligned}$$

For Question # 14:

- Which is bigger? 7/24 or 17/60? [Answer: 7/24.]
- Which is bigger? 5/18 or 3/10? [Answer: 3/10.]
- Which is bigger? 8/15 or 11/20? [Answer: 11/20.]
- Which is bigger? 9/14 or 13/21? [Answer: 9/14.]

For Questions # 15 & # 16:

- The shipment will weigh 93.775 tons. You can also write 93.7750 tons, since we often use six significant digits in this textbook.
- There will be $1419.37\cdots \approx 1419$ people in South Dakota, and $4369.81\cdots \approx 4370$ people in Brooklyn, with bunions severe enough to warrant surgery.
- If you did North Dakota by accident, instead of South Dakota, there will be $1191.61\cdots \approx 1192$ people.

For Question # 17:

- We get $6.392\times 10^{-3} = 0.006392$.
- We get $4.502\times 10^{-4} = 0.0004502$.
- We get $7.932\times 10^{-2} = 0.07932$.
- We get $1.918\times 10^{-1} = 0.1918$.

I'm afraid that if you got 2 or more of these wrong, then you should consider the entire question wrong. If you got 1 or 0 of these wrong, then you can consider the entire question correct. That's because this is very fundamental material.

For Question # 18:

- We get $8.613 \times 10^6 = 8,613,000$.
- We get $3.016 \times 10^8 = 301,600,000$.
- We get $265 \times 10^7 = 2,650,000,000$.
- We get $4.57801 \times 10^4 = 45,780.1$.

As before, if you got 2 or more of these wrong, then you should consider the entire question wrong. If you got 1 or 0 of these wrong, then you can consider the entire question correct. That's because this is very fundamental material.

For Question #19.

- What is the 2nd significant digit of 28,900,000,000? [Answer: 8.]
- What is the 3rd significant digit of 0.0004236? [Answer: 3.]
- What is the 4th significant digit of 352.691? [Answer: 6.]
- What is the 5th significant digit of 24,879,000? [Answer: 9.]
- What is the 6th significant digit of 0.006700249? [Answer: 4.]

Yet again, if you got 2 or more of these wrong, then you should consider the entire question wrong. If you got 1 or 0 of these wrong, then you can consider the entire question correct. That's because this is very fundamental material.

Question # 1 and Question # 2 cover some skills that are actually covered by another diagnostic—Diagnostic Zero. If you found these questions at all difficult, especially if you got more than one wrong, then you should take Diagnostic Zero as soon as possible. Even if you only had mild difficulty with Questions 1 & 2, please try Diagnostic Zero to set your mind at ease. Give yourself an hour or half-hour to rest and relax, and then turn to Diagnostic Zero.

On the other hand, most students will have no problem with Question # 1 and Question # 2, so for them, there is no reason to look at Diagnostic Zero.

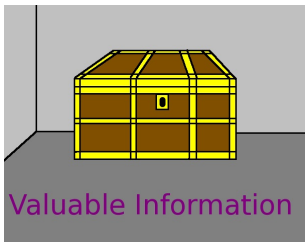
- If you got Question # 3 wrong, please read Module 1.2: "Using the Quadratic Formula."
- If you got Question # 4 or # 5 wrong, please read Module 1.3: "Rapidly Graphing Lines."
- If you got Question # 6, # 7, or # 8 wrong, then please read Module 1.4: "Intersecting Two Lines (Part One)" and "1.5: Intersecting Two Lines (Part Two)."
- If you got Question # 15 or Question # 16 wrong, then please study Module 1.1: "Direct Proportions" in detail.
- If you got Questions # 17, # 18, or # 19 wrong, the please study Module 0.1: "Scientific Notation and Significant Figures."

The list continues in the next box.

- If you got Question # 9, # 10, or # 11 wrong, then please read Module 1.6: "Working with Functions" and Module 1.7: "Multiplying and Squaring Polynomials or Functions."
- However, if you only got the multiplications wrong on # 9, # 10, or # 11 then you can just read Module 1.7: "Multiplying and Squaring Polynomials or Functions" and skip "1.6: Working with Functions." This means that you got either 8c and/or 9c wrong, but that 8a, 8b, 9a, and 9b were all correct.
- While all algebra skills are important, the least important module in Chapter 1 is Module 1.7: "Multiplying and Squaring Polynomials or Functions." Therefore, don't be alarmed if you found that to be too difficult.

If you got Question # 12, # 13, or # 14 wrong, then you should be happy to know that Khan Academy has a plethora of videos available for fractions. Many students have issues with this material, since it is from so long ago in their education. There is even a portal (available at the following link) where all the videos are housed.

`https://www.khanacademy.org/math/arithmetic/fractions`



If you got Question # 14 wrong, then you should also note that some students find it easier just to convert both fractions to decimals. Then it is easy to figure out which is the larger number and which is the smaller number.

Mathematically, the purpose of this question was to remind you that fractions can only be compared if they have common denominators. Pragmatically speaking, decimals are much easier to compare.

That's it for Diagnostic Two. Thank you for sitting through this rather dry, but very important, experience.