Worksheet on Direct Proportions

Gregory V. Bard

January 19, 2017

Questions

1. A company that manufactures toys has a new product, but the marketing department was unsure of how well the new product would sell, so they made the initial run very small. The first shipment was 80,000 toys but sold extremely well. As a result, the new shipment is 620,000 toys. The old shipment weighed 12.1 metric tons. How much will the new shipment weigh? (Clearly the weight of the shipment is directly proportional to the number of toys.)

2. Continuing with the previous problem, sending the first shipment from China to the USA cost $ 44,000 in cargo shipping fees. How much should sending the second shipment cost? (It is very safe to assume that the cost of the shipping is directly proportional to the number of toys.)

3. Cindy has decided to volunteer with the Peace Corps and has been sent to some isolated villages in an remote province of a sub-Saharan Africa nation. There, she and some other volunteers are establishing clean running water for those villages. One day, she is delivering a truck load of small disposable water filters to a village that does not have clean water yet, and has a long conversation over lunch with a local doctor about water issues.

   It seems that many more water filters could be rather helpful and Cindy's parents had expressed interest in making a financial donation to support her work. Accordingly, Cindy asks how many water filters could be put to use. The provincial doctor says that they could most certainly make effective use of half a million disposable water filters. She texts her parents and they place an order with the manufacturer of the water filters, as well as for the cargo service of their favorite airline, to ship the crates to the airport near the capital city. Unfortunately, the challenging part is getting the water filters from the airport to the remote province, which is 487 miles away.

   A nearby army base offers to lend as many pickup trucks as would be required, but Cindy doesn't know how many trucks to request. The doctor says that a previous shipment of 12,500 filters filled up 2 army pickup trucks almost exactly. With this information, she opens up the calculator app on her smart phone, and computes the number of pickup trucks required. How many trucks is that?

Citation: The following computation was inspired by the article "Voters In Wyoming Have 3.6 Times The Voting Power That I Have. It's Time To End The Electoral College," by William Petrocelli, published in *The Huffington Post* on November 10th, 2016.

4. The voting power of US states in the electoral college is not awarded by proportion to population, which we can mathematically prove. The technical term for this is *disproportionate*. We're going to explore that point now, in detail.

   The population of Wyoming in the 2010 census was 563,767 and the population of California in the 2010 census was 37,254,503. California is worth 55 votes in the electoral college, and Wyoming is worth 3 votes in the electoral college. (If you're wondering why I am using 2010 data, that's because the next census after 2010 will be in the year 2020, and that data is not yet available.)

   If it were to be desired to award electoral votes proportionately, then how many votes would California's population have to be given, in order to be in proportion with Wyoming?

5. A high-tech biomedical-research firm has developed a new product for customers that have Type I diabetes (that's the type that either you are born with or born without, as compared to Type II diabetes which can start at any point in a patient's life.) A marketing study was done on 10,000 randomly selected people in the United States. Of those, 528 of them said that they would buy this new gadget that your company is selling. This is understandable because most people simply do not have diabetes of either type.

   Two of your friends work in the sales department of your company: Joel, covering New York City, and Mike, covering the state of Wisconsin. Knowing that there are roughly 5,712,000 people in Wisconsin, and roughly 8,175,000 people in New York City, how many gadgets can Joel and Mike expect to sell?

Note: It was legitimate to use a proportion in the previous problem, because the illness being discussed is one that is caused genetically, and occurs (somewhat) randomly throughout the population. Therefore, we can assume that the fraction of people in Wisconsin, New York City, and the nation as a whole, is a constant.

6. A medical study is done on a sample of 20,350 people. It discovers that 413 of them have a particular dangerous condition. If the sample was a good representative sample of the world population, the number of people in the world with that dangerous condition should be proportional. Having read

this, a manager wants to know how many people in the world might have that dangerous medical condition. She asks her intern, who pulls up the world population clock on his smart phone. He notes that the number of people in the world in July of 2009 was roughly 6,790,062,216, (according to the CIA World Fact Book, 2009 edition). Estimate the number of people in the world with the condition in July of 2009.

Discussion: Let's consider now two cases where direct proportions would not be a good model at all for predicting the number of customers.

- Let's imagine that you are in the business of marketing blue jeans. According to the August 16, 2012 article by Rachel Pomerance "Most and Least Obese U.S. States" published in *US News and World Report*, it turns out that obesity does vary by geographic location. In Mississippi 34.9% of the population was obese, where is Colorado, 20.7% of the population was obese. (Note, the word "obese" as a precise medical definition: if the person's Body Mass Index is greater than or equal to 30, then the person is obese; if the BMI is less than 30, then the person is not obese—though some medical doctors and mathematicians find this measurement to be oversimplified.)
- In any case, do you think the sizes of blue jeans sold, or the number of bikinis sold, in Mississippi versus Colorado, would be easily comparable? Of course not!
- Consider the sale of snowblowers. Do you think the number of people who want to buy a snowblower in Wisconsin will be in direct proportion to the number of people who want to buy a snowblower in Florida?
- The bottom line is that demand, particularly consumer demand, is notoriously hard to compute. Businesspeople have come up with a tool to get around this roadblock, called "break even analysis," which we will study later in the textbook.
- There are other circumstances when you cannot use a direct proportion, but we'll study that more when we learn about "Non-Linear Proportions."

Historical Note: Archeological evidence reveals that direct proportions have been known to the ancients. However, their financial, industrial, economic, and commercial applications were widely popularized by the Franciscan monk Luca Pacioli (1445–1517) in his book *Summa de arithmetica, geometria, proportioni et proportionalit*. This book is more famous for introducing "the double-entry system," which is the foundation of modern systems of accounting. He also invented the "Rule of 72" which you will learn about later in the textbook.

Luca Pacioli had positions teaching and tutoring mathematics throughout Renaissance Italy. When writing a different book about perspective in art and the golden ratio, titled *De divina proportione*, he had one of his students draw the diagrams for him. Those extremely lovely diagrams made the book famous, and that lad is known to us today as Leonardo da Vinci (1452–1519).

7. Jeremy is baking some muffins and he has an ample supply of ingredients except for butter. The recipe calls for 3/4ths of a cup of butter and he has sticks of butter equivalent to $7\frac{1}{2}$ cups in the fridge. The recipe produces 24 muffins. Jeremy wants to bake as many muffins as possible, for a bake sale at the elementary school of his nephew and niece.
   - How many muffins can he make?
   - If the recipe calls for 1/3 cups of milk, how much milk will he use in total?
8. A recipe serves 4 people, and calls for 14 ounces of tomato sauce, and you're going to be serving 7 people. How much tomato sauce should you use?
9. I have a package of organic blackberries, and it says 170 grams, but it doesn't say how many calories are in the package. A quick internet search reveals that one serving of blackberries has 144 grams and is 62 calories. How many calories do I consume if I eat the entire package of blackberries?
10. Now I have a package of organic raspberries, also 170 grams, and again the number of calories is not listed. A quick internet search reveals that one serving of blackberries has 123 grams and is 65 calories. How many calories do I consume if I eat the entire package of blackberries?
11. A map has a scale that shows how large 50 miles would be on the map. Two towns are shown and you measure that they are 7 inches apart, and then you measure the scale, and discover 50 miles is roughly 2 1/4 inches. How far apart are the two towns?
12. A map you are holding says "Scale 240,000:1." The distance between two intersections on the map looks like a quarter of an inch. How many feet are the two intersections apart in real life? Reminder: 12 inches is one foot.
13. A very old trick is used to measure the distance to a lightening strike in a thunderstorm. One simply counts the number of seconds between the lightening and the thunder, and that's the distance in thousands of feet, very roughly. However, this technique is folklore, not science.

    From a scientific point of view, It is true that the distance and the time are proportional. Let us imagine that you're in a thunderstorm, and you hear thunder 5 seconds after lightening. You check a scientific website, which tells you that a 12-second gap means 13,050 feet away.

How far away, then, would be the lightening strike that you just heard, from a scientific point of view? How far away, according to the folklore trick?

14. At a large company banquet, two corporate representatives are seated with 10 newly hired salespeople around a large round table. Five of the salespeople have each been given one of the boroughs of New York City as their territory, and the other five have each been given an entire state. They are going to be selling surgical devices to help treat a painful type of bunion. The salespeople are curious how large their markets will be. The senior corporate representative says he has an email from the Delaware rep saying that there is an estimated 1562 people in Delaware with bunions severe enough to require surgery. He then tasks the junior corporate rep to compute the market for the ten salespeople seated around the table. The junior corporate rep does a quick internet search, and finds the following population data:

Wyoming	576,412	Staten Island	470,728
Vermont	626,011	The Bronx	1,408,473
North Dakota	699,628	Manhattan	1,619,090
Alaska	731,449	Queens	2,272,771
South Dakota	833,354	Brooklyn	2,565,635
Delaware	917,092		

This data represents July 1st, 2012 estimates.

(For the sake of this problem, you can assume that the condition occurs randomly enough that a direct proportion is appropriate.)

For now, just compute the size of the market for Brooklyn and for Wyoming. In the answers section, we will describe a great short cut, called the "scaling factor" method.

15. Now use the scaling factor method and the data in the previous problem to calculate the size of the market in Queens, and in Vermont.

Note: You might be surprised to see such small numbers for New York City. Actually, for any city, three numbers for population can be used. The first is the legal population: how many people actually reside within the strict boundaries of the city, vote for the mayor, and write the name of the city on their census bureau forms. The second is the metropolitan statistical area (MSA), which includes suburbs, and includes many people who work in the city but live outside it. The third, the combined statistical area (CSA), includes nearby secondary towns, possibly as far as an hour away, which would include people who might shop or use services in the city frequently even if they don't visit every day.

For most purposes, the MSA is the number that makes sense. Each of the following examples uses data from the 2010 Census. For example, the population of the New York City Metropolitan area is 19,567,410, as compared to a legal population of only 8,175,133. Another example is Washington, DC, whose metropolitan area has 5,860,342 people, even though only 601,723 people actually reside in "the District of Columbia" that puts the "D" and the "C" in "DC." Yet another melodramatic example is Chicago, whose metropolitan area has 9,461,105 people, even though the legal population is 2,695,598.

16. Now, recompute the solutions to Problems 1 and 2 using the scaling factor method.

17. Next, recompute the solutions to Problems 3 and 4 using the scaling factor method.

18. America's major cities are actually rather different from each other. The problem we are about to solve is going to explain how Detroit and New York City are very different, and in particular, how the murder rates are extremely different. This problem might be confusing for some students. However, I think it is very beneficial to labor through it, perhaps several times if needed. I feel that way because one cannot truly understand mathematical models unless one knows when to reject them. We're going to see what the numbers look like if we assume that the crime rates are proportional. Using those numbers, we will then reject the notion that the crime rates are proportional.

    - In 2009, there were 386 homicides[1] in New York City.
    - The population[2] of New York City in 2009 was 8,391,881.
    - The population[3] of Detroit in 2009 was 821,792.
    - In 2009, I have conflicting data[4] for the number of homicides in Detroit. The numbers I have are 364, 361, and 286. Therefore, let's go with the median figure, 361.

    We can pose the following two small questions using proportions, and then we will be able to very safely answer the bigger question.

    (a) If Detroit had a homicide rate proportional to New York City, how many homicides would there have been during 2009?

[1] http://projects.nytimes.com/crime/homicides/map

[2] http://www.health.ny.gov/statistics/chac/population/newyorkcity.htm

[3] http://real-estate-and-urban.blogspot.com/2011/03/to-finish-previous-posts-thought.html

[4] http://policelink.monster.com/news/articles/148752-detroit-homicides-hit-lowest-level-since-1967
http://www.ci.detroit.mi.us/DepartmentsandAgencies/PoliceDepartment/CrimeStatistics.aspx
http://champage.blogspot.com/2011/11/2009-detroit-homicidesmurders-list.html

(b) If New York City had a homicide rate proportional to Detroit, how many homicides would there have been during 2009?

(c) How does the risk of homicide in Detroit compare to the risk of homicide in New York City?

Note: The accept/reject decision-making process is sometimes called "critical thinking" by educators who wish to sound trendy.

19. Sarah is 17 weeks pregnant. Her doctor tells her that by this time, the baby's fingers and toes have finger nails and toe nails. She is curious how large (or I should say, how small) the baby's toenails would be. The baby is 5 inches long, while Sarah is 5'6" tall. She assumes that the length of the toenail is directly proportional to the height of the person, and this is roughly correct. She measures her own toenail to be 22 mm of an inch long on the big toe. Approximately how many millimeters is the baby's toenail on the big toe?

20. Continuing with the previous problem, If Sarah's index finger is 7.6 centimeters long, how long is the baby's index finger?

21. If you've ever had the chance to swim deep underwater either in a swimming pool or scuba diving, then you've noticed that there is a great deal of pressure as you go deeper underwater. As it turns out, the additional pressure underwater is directly proportional to depth.

    A convenient unit of pressure is "1 atmosphere." For example, if we say that the pressure is "2 atmospheres" at some depth, then that means there is twice as much pressure at that depth than at the surface.

    Given that 338 feet below water is 10 atmospheres of additional pressure...

    (a) ...how many atmospheres of additional pressure is there at the bottom of the deep-end of a swimming pool, if the depth is marked there as 12 ft?

    (b) ...how many atmospheres of additional pressure are there on a submarine operating 600 ft deep?

    (c) ...how many atmospheres of additional pressure are there at the bottom of the Mariana Trench, which is 36,000 feet deep?

Warning: Some students have, in the past, found the following long problem confusing. It can be safely skipped.

22. Macroeconomic statistics take time to calculate because the data sets can be enormous. For example, the Gross Domestic Product of a state or country is the cash value of all goods and services produced in that state or country. This can range from mining to finance, from agriculture to entertainment, from the military to education, from construction to utilities, and so forth. However, sometimes the information is needed before

data collection can be completed. A state government, however, knows how much sales tax is coming in each month. This can allow them to predict the GDP using a direct proportion. Let's imagine that you're an accounting intern in some bureau of the Kansas state government.

- Let's say it is early January in 2009. The GDP of 2007 would surely be known by now, and the sales tax data of both 2007 and 2008 would be complete. Using the sales tax data of 2007 and 2008, and the GDP for 2007, what would your estimate of the GDP for 2008 be?
- Now, using the actual GDP for 2008, what is the relative error of that estimate?

Sales Tax, 2011	2,347,193,366.93	GDP in 2011	134,767 million
Sales Tax, 2010	2,075,193,412.55	GDP in 2010	126,640 million
Sales Tax, 2009	1,866,223,078.37	GDP in 2009	121,967 million
Sales Tax, 2008	1,990,655,687.51	GDP in 2008	124,330 million
Sales Tax, 2007	1,917,547,819.47	GDP in 2007	120,599 million

By the way, in case you have forgotten it, the formula for relative error is:

$$\text{relative error} = \frac{\text{estimate} - \text{truth}}{\text{truth}}$$

23. Now repeat the above problem for future years:

- Using the sales tax data of 2008 and 2009, and the GDP for 2008, what would your estimate of the GDP for 2009 be? What is the relative error of that estimate?
- Using the sales tax data of 2009 and 2010, and the GDP for 2009, what would your estimate of the GDP for 2010 be? What is the relative error of that estimate?
- Using the sales tax data of 2010 and 2011, and the GDP for 2010, what would your estimate of the GDP for 2011 be? What is the relative error of that estimate?

Note: Anyone interested in the sources of data used here can check the following:

- For the GDP dollar values, look at `http://www.ipsr.ku.edu/ksdata/ksah/business/7gsp2.pdf`
- For the sales tax information, look at State Sales Tax Collections by County `http://www.ksrevenue.org/salesreports.html#state`

Note: Some might say that I technically should have said GSP or "gross state product," because GDP or "gross domestic product" is reserved for nations, not states. However, the use of the term GSP is both old-fashioned and confusing. Many official US Government documents use GDP in reference to states. In any case, either term refers to the total market value of goods and services made inside some state or nation during that year.

Answers

1. The proportion should be

$$\begin{array}{ccccc} \text{“first shipment”} & \text{vs.} & \text{“second shipment”} & & \\ \dfrac{12.1}{80,000} & = & \dfrac{w}{620,000} & & \dfrac{\leftarrow \text{“tonnage”}}{\leftarrow \text{“toys”}} \end{array}$$

which we can solve by multiplying both sides by 620,000 to obtain

$$w = 620,000 \frac{12.1}{80,000} = 93.775 \text{ metric tons}$$

2. Actually, this problem could be solve two ways. First, by the number of toys, in which case the proportion should be

$$\begin{array}{ccccc} \text{“first shipment”} & \text{vs.} & \text{“second shipment”} & & \\ \dfrac{44,000}{80,000} & = & \dfrac{c}{620,000} & & \dfrac{\leftarrow \text{“cost”}}{\leftarrow \text{“toys”}} \end{array}$$

which we can solve by multiplying both sides by 620,000 to obtain

$$c = 620,000 \frac{44,000}{80,000} = \$\ 341,000$$

Alternatively, we could have made the proportion by the weights, in which case the proportion should be

$$\begin{array}{ccccc} \text{“first shipment”} & \text{vs.} & \text{“second shipment”} & & \\ \dfrac{44,000}{12.1} & = & \dfrac{c}{93.775} & & \dfrac{\leftarrow \text{“cost”}}{\leftarrow \text{“weight”}} \end{array}$$

which we can solve by multiplying both sides by 93.775 to obtain

$$c = 93.775 \frac{44,000}{12.1} = \$\ 341,000$$

Reassuringly, both approaches give exactly the same answer.

Theory: The fact that both approaches give the same answer is not a coincidence. If the number of toys is directly proportional to the weight, and then number of toys is directly proportional to the shipping cost, then it must be true that the weight is directly proportional to the shipping cost. More generally, if “Property A” is directly proportional to “Property B” and “Property A” is directly proportional to “Property C” then “Property B” is directly proportional to “Property C.”

3. The proportion should be

$$\begin{array}{ccc} \text{“new shipment”} & \text{vs.} & \text{“small shipment”} \end{array}$$

$$\frac{j}{500,000} = \frac{2}{12,500} \qquad \frac{\leftarrow \text{“jeeps”}}{\leftarrow \text{“disposable filters”}}$$

Now we have

$$500,000\frac{2}{12,500} = \frac{1,000,000}{12,500} = 80$$

and so she requests 80 pickup trucks. For even a small military base, this should be no problem.

Did you notice that we made no use of the fact that it is 487 miles from the airport to the remote province? That number was not relevant to the problem, and so therefore, we did not use it. In real world situations, you often have far more information than you actually need.

4. First, we write the direct proportion:

$$\begin{array}{ccc} \text{“population”} & \text{vs.} & \text{“electoral college vote”} \end{array}$$

$$\frac{37,254,503}{563,767} = \frac{x}{3} \qquad \frac{\leftarrow \text{“California”}}{\leftarrow \text{“Wyoming”}}$$

Then we have

$$\begin{aligned} (37,254,503)(3) &= 563,767x \\ 111,763,509 &= 563,767x \\ 198.244\cdots &= x \end{aligned}$$

which means that California would have to be given 198 or 199 electoral votes, instead of the 55 that they currently have. On the basis of this, one journalist (see below) computed that the voting power of a Wyoming resident was 3.6× as much as the voting power of a California resident.

You can read more in the article “Voters In Wyoming Have 3.6 Times The Voting Power That I Have. It’s Time To End The Electoral College,” by William Petrocelli, published in *The Huffington Post* on November 10th, 2016.

5. The proportion for Wisconsin is

$$\begin{array}{ccc} \text{“market study”} & \text{vs.} & \text{“New York City”} \end{array}$$

$$\frac{528}{10,000} = \frac{s}{5,712,000} \qquad \frac{\leftarrow \text{“sales”}}{\leftarrow \text{“population”}}$$

So the project sales for Michael would be

$$s = 5,712,000\frac{528}{10,000} = (571.2)(528) = 301,593.\cdots \text{ customers.}$$

Meanwhile, the proportion for New York City would be

$$\begin{array}{ccc} \text{"market study"} & \text{vs.} & \text{"New York City"} \end{array}$$

$$\frac{528}{10,000} = \frac{s}{8,175,000} \qquad \frac{\leftarrow \text{"sales"}}{\leftarrow \text{"population"}}$$

So the project sales for Joel would be

$$s = 8,175,000\frac{528}{10,000} = (817.5)(528) = 431,640 \text{ customers.}$$

6. The proportion should be

$$\begin{array}{ccc} \text{"has condition"} & \text{vs.} & \text{"population"} \end{array}$$

$$\frac{x}{413} = \frac{6,790,062,216}{20,350} \qquad \frac{\leftarrow \text{"the world"}}{\leftarrow \text{"that study"}}$$

And so we obtain

$$x = 413\frac{6,790,062,216}{20,350} = 137,803,228.\cdots$$

(Really we should round to the nearest thousand, because nine digits of precision is not very realistic. We would then write 137,803,000 or 137,804,000.)

7. Here, we have two circumstances, the "recipe" and "Jeremy." Also, we have two quantities which are "butter" and "muffins." We would set up the following proportion.

$$\begin{array}{ccc} \text{"butter"} & \text{vs.} & \text{"muffins"} \end{array}$$

$$\frac{7.5}{3/4} = \frac{n}{24} \qquad \frac{\leftarrow \text{"Jeremy"}}{\leftarrow \text{"the recipe"}}$$

Then we would solve for n with

$$n = 24\frac{7.5}{3/4} = (24)(10) = 240 \text{ muffins}$$

Next, we can tackle the milk. The proportion for the milk could be built on a "butter vs. milk" perspective or a "muffins vs. milk" perspective. If we choose to build it with butter

$$\begin{array}{ccc} \text{“butter”} & \text{vs.} & \text{“milk”} \end{array}$$

$$\frac{7.5}{3/4} = \frac{m}{1/3} \qquad \frac{\leftarrow \text{“Jeremy”}}{\leftarrow \text{“the recipe”}}$$

Then we would solve for m with

$$m = \frac{1}{3} \times \frac{7.5}{3/4} = (1/3)(10) = 10/3 \text{ cups} = 3\frac{1}{3} \text{ cups}$$

Alternatively, we could use

$$\begin{array}{ccc} \text{“muffins”} & \text{vs.} & \text{“milk”} \end{array}$$

$$\frac{240}{24} = \frac{m}{1/3} \qquad \frac{\leftarrow \text{“Jeremy”}}{\leftarrow \text{“the recipe”}}$$

Then we would solve for m with

$$m = \frac{1}{3} \times \frac{240}{24} = (1/3)(10) = 10/3 \text{ cups} = 3\frac{1}{3} \text{ cups}$$

Either way, Jeremy will produce 240 muffins and use up $3\frac{1}{3}$ cups of milk.

8. The proportion should be

$$\begin{array}{ccc} \text{“sauce”} & \text{vs.} & \text{“people”} \end{array}$$

$$\frac{t}{14} = \frac{7}{4} \qquad \frac{\leftarrow \text{“You”}}{\leftarrow \text{“the recipe”}}$$

Therefore, we have

$$t = 14\frac{7}{4} = 24.5 = 24\frac{1}{2} \text{ ounces of tomato sauce}$$

9. The proportion should be

$$\begin{array}{ccc} \text{“internet”} & \text{vs.} & \text{“my package”} \end{array}$$

$$\frac{62}{144} = \frac{c}{170} \qquad \frac{\leftarrow \text{“calories”}}{\leftarrow \text{“grams”}}$$

Therefore, we have

$$c = 170\frac{62}{144} = 73.194\overline{4}$$

or about 73.2 calories.

10. The proportion should be

$$\begin{array}{ccc} \text{“my package”} & \text{vs.} & \text{“the internet”} \end{array}$$

$$\frac{c}{170} = \frac{65}{123} \quad \frac{\leftarrow \text{“calories”}}{\leftarrow \text{“grams”}}$$

Therefore, we have

$$c = 170\frac{65}{123} = 89.8373\cdots$$

or about 89.8 calories.

11. The proportion should be

$$\begin{array}{ccc} \text{“real life”} & \text{vs.} & \text{“on the map”} \end{array}$$

$$\frac{d}{50} = \frac{7}{2.25} \quad \frac{\leftarrow \text{“between towns”}}{\leftarrow \text{“the scale”}}$$

Therefore, we have

$$d = 50\frac{7}{2.25} = 155.5\overline{5} \text{ miles}$$

12. The scale tells us that 240,000 inches in real life is 1 inch on the map. Therefore, we can write the following proportion:

The proportion should be

$$\begin{array}{ccc} \text{“real life”} & \text{vs.} & \text{“on the map”} \end{array}$$

$$\frac{d}{240,000} = \frac{1/4}{1} \quad \frac{\leftarrow \text{“the intersection”}}{\leftarrow \text{“the scale”}}$$

Therefore, we have

$$d = 240,000\frac{1/4}{1} = 60,000 \text{ inches} = \frac{60,000}{12} \text{ feet} = 5000 \text{ feet}$$

Note: Here would go the explanation of scaling factors.

13. The proportion should be

$$\begin{array}{ccc} \text{“the field guide”} & \text{vs.} & \text{“your storm”} \end{array}$$

$$\frac{13,050}{12} = \frac{f}{5} \quad \frac{\leftarrow \text{“feet away”}}{\leftarrow \text{“gap in seconds”}}$$

Then we would get

$$f = 5\frac{13,050}{12} = 5437.50 \text{ feet}$$

As you can see, the old folklore trick isn't so bad, because it would have predicted 5000 feet!

14. If the junior rep has to calculate the answer for ten people, during a business lunch, it wouldn't be much fun to actually grind through the calculation with the same method as we've done all of our calculations up until this point. One option is to use a spreadsheet to do the calculations rapidly. There is also another shortcut, called "a scaling factor."

    - Brooklyn is $\frac{2,565,635}{917,092} = 2.79757\cdots$ times as large as Delaware.
    - The Brooklyn market then will be $(1562)(2.79757\cdots) = 4369.81\cdots$ or about 4370 patients.
    - Wyoming is $\frac{576,412}{917,092} = 0.628521\cdots$ times as large as Delaware.
    - The Wyoming market then will be $(1562)(0.628521\cdots) = 981.750\cdots$ or about 982 patients.
    - The phrase "scaling factor" refers to quantity "2.79757" or "0.628521." Basically, it is how many times bigger Situation B is compared to Situation A. If Situation B is larger than Situation A, then the scaling factor is greater than one. If Situation B is smaller than Situation A, then the scaling factor is less than one. If the situations are equal (which would make the problem pointless) then the scaling factor equals one.

15. This problem is in two parts:

    - Queens is $\frac{2,272,771}{917,092} = 2.47823\cdots$ times as large as Delaware.
    - The Queens market then will be $(1562)(2.47823\cdots) = 3871.00\cdots$ or about 3871 patients. By the way, there's an odd thing that just happened. Just after the second 0 is a 5. In other words, with more precision the answer is $3871.00563\cdots$. It is important to not fret over, or worry about, the least significant digit of any number that you write down.

Note: Later, when you take a course in statistics, you'll learn how to compute the uncertainty on calculations of this type. It turns out that you can compute 3871 ± 62 in this case. However, that's not for us to worry about right now.

    - Vermont is $\frac{626,011}{917,092} = 0.682604\cdots$ times as large as Delaware.
    - The Vermont market then will be $(1562)(0.682604\cdots) = 1066.22\cdots$ or about 1066 patients.

16. This problem is in two parts.

- In Problem 1: First, the scaling factor is $\frac{620{,}000}{80{,}000} = 7.75$. This means that the second order is 7.75 times as large as the first order.
- In Problem 1: Second, the weight is $(12.1)(7.75) = 93.775$ metric tons.
- In Problem 2: Again, the scaling factor is $\frac{620{,}000}{80{,}000} = 7.75$. Once again the second order is 7.75 times as large as the first order.
- In Problem 2: Second, the shipping cost is $(44,000)(7.75) = 341,000$ dollars.

17. This problem is also in two parts.

- In Problem 3: First, the scaling factor is $\frac{500{,}000}{12{,}500} = 40$. This means that the new shipment is 40 times as large as the old shipment.
- In Problem 3: Second, the number of trucks required is $(2)(40) = 80$.
- In Problem 4: First, the scaling factor is $\frac{1.99065}{1.91754} = 1.03812\cdots$ which means that the sales tax revenues in 2008 were 3.812% higher than in 2007. Sounds reasonable.
- In Problem 4: Second, the GDP prediction is $(120,559)(1.03812\cdots) = 125,155.\cdots$ million dollars, or more succinctly, 125.155 billion dollars.

18. This question is divided into three parts

(a) The proportion should be

$$\begin{array}{cccc} \text{“New York”} & \text{vs.} & \text{“Detroit”} & \\ \\ \dfrac{386}{8,391,881} & = & \dfrac{h}{821,792} & \dfrac{\leftarrow \text{“homicides”}}{\leftarrow \text{“population”}} \end{array}$$

Enabling us to solve for h by

$$h = 821,792 \frac{386}{8,391,881} = 37.7998\cdots$$

Therefore, if Detroit had the homicide rate of New York City, then there would have been 37 or 38 homicides during 2009 instead of the actual 361 victims.

(b) The proportion should be

$$\begin{array}{cccc} \text{“New York”} & \text{vs.} & \text{“Detroit”} & \\ \\ \dfrac{h}{8,391,881} & = & \dfrac{361}{821,792} & \dfrac{\leftarrow \text{“homicides”}}{\leftarrow \text{“population”}} \end{array}$$

Enabling us to solve for h by

$$h = 8,391,881 \frac{361}{821,792} = 3686.41\cdots$$

Therefore, if New York City had the homicide rate of Detroit, then there would have been 3687 or 3686 homicides during the year instead of the actual 386 victims.

(c) Based on this evidence, it is phenomenally obvious that the risk of being a homicide victim is far higher in Detroit than in New York City.

19. We must be careful to realize that 5'6" tall actually means 60" tall. This can be found by

$$5'6'' = 5 \text{ feet and } 6 \text{ inches} = 5\times 12 \text{ inches}+6 \text{ inches} = 60+6 \text{ inches} = 66 \text{ inches}$$

Now the proportion is

$$\begin{array}{ccc} \text{“Sarah”} & \text{vs.} & \text{“the baby”} \\ \\ \dfrac{22}{66} & = & \dfrac{T}{5} \qquad \dfrac{\leftarrow \text{“toe-nail size in mm”}}{\leftarrow \text{“height in inches”}} \end{array}$$

Next, we obtain

$$T = 5\frac{22}{66} = 1.66\overline{6} \text{ mm}$$

and so the baby's toenail is about 1.66 mm long, which is very small. Also, note that we didn't convert anything from US units into metric, nor did we convert anything in metric to US units. There is simply no reason to waste time doing that.

20. Similarly to the previous problem, we have

$$\begin{array}{ccc} \text{“Sarah”} & \text{vs.} & \text{“the baby”} \\ \\ \dfrac{7.6}{66} & = & \dfrac{f}{5} \qquad \dfrac{\leftarrow \text{“finger-size in cm”}}{\leftarrow \text{“height in inches”}} \end{array}$$

Next we have

$$f = 5\frac{7.6}{66} = 0.57\overline{57} \text{ cm}$$

and so the baby's index finger is about 0.57 mm long, which is very small. As before, note that we didn't convert anything from US units into metric, nor did we convert anything in metric to US units.

21. This question is in three parts:

(a) The proportion should be

"given example" vs. "swimming pool"

$$\frac{10}{338} = \frac{P}{12} \qquad \frac{\leftarrow \text{"pressure (in atmospheres)"}}{\leftarrow \text{"depth (in feet)"}}$$

which gives $P = 0.355029\cdots$ additional atmospheres of pressure, or about 1.35 atmospheres total.

(b) The proportion should be

"given example" vs. "submarine'

$$\frac{10}{338} = \frac{P}{600} \qquad \frac{\leftarrow \text{"pressure (in atmospheres)"}}{\leftarrow \text{"depth (in feet)"}}$$

which gives $P = 17.7514\cdots$ additional atmospheres of pressure, or about 18.75 atmospheres total.

(c) The proportion should be

"given example" vs. "Mariana Trench"

$$\frac{10}{338} = \frac{P}{36,000} \qquad \frac{\leftarrow \text{"pressure (in atmospheres)"}}{\leftarrow \text{"depth (in feet)"}}$$

which gives $P = 1065.08\cdots$ additional atmospheres of pressure, or about 1066.08 atmospheres total.

22. The proportion should be

"The Year 2008" vs. "The Year 2007"

$$\frac{G}{1.99065\cdots} = \frac{120,559}{1.91754\cdots} \qquad \frac{\leftarrow \text{"GDP in millions"}}{\leftarrow \text{"sales tax in billions"}}$$

Then we have

$$G = 1.99065\cdots \frac{120,559}{1.91754\cdots} = 125,155.\cdots$$

thus we can conclude, for the first part, that GDP will be around 125,155 million dollars, or as would be more commonly spoken, 125.155 billion dollars.

The relative error is given by

$$\frac{\text{estimate} - \text{truth}}{\text{truth}} = \frac{125,155 - 124,330}{124,330} = \frac{825}{124,330} = 0.00663556\cdots \approx 0.66\%$$

Therefore, for the second part, the relative error is $+0.00663556\cdots$ or $+0.66\%$. This is an outstanding estimate, extremely close to the truth.

23. This problem is in three parts, each to be solved very similarly to the previous problem. Therefore we omit the details.

- The estimate is $116,558.\cdots$ million. The relative error is $-0.0443480\cdots$ or -4.43%.
- The estimate is $135,624.\cdots$ million. The relative error is $+0.0709412\cdots$ or +7.09%.
- The estimate is $143,238.\cdots$ million. The relative error is $+0.0628638\cdots$ or +6.28%.

Note: Did you notice how our estimates weren't very accurate even though we had 12 digits of precision in the sales tax data? Having all those digits gives us precision, but it doesn't give us accuracy. Accuracy is how far from the truth an estimate might happen to be. Precision, in mathematics, is a reflection of the number of decimal places reported. Just because a number has many decimal places does not mean that it is trustworthy!