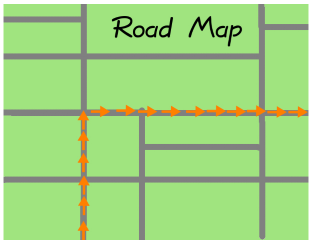
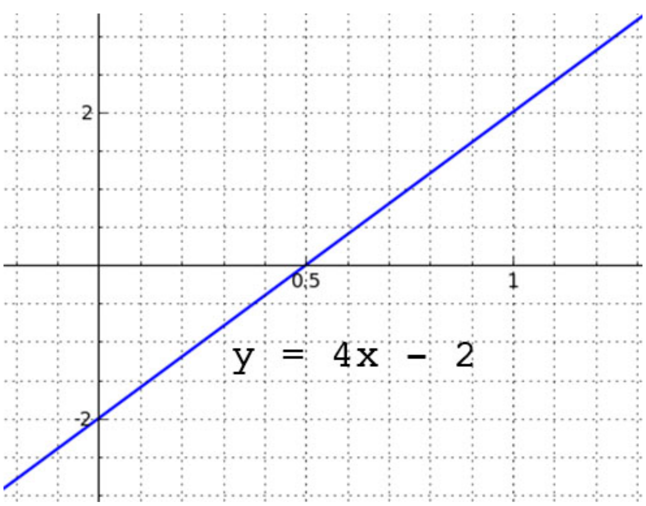
Module 1.3: Intercepts and Rapidly Graphing Lines



This module will review the techniques of rapidly graphing a line on a coordinate plane. By "graphing" I of course mean physically putting pencil to paper and drawing a straight line that accurately represents a linear equation in two variables, or equivalently, a linear function. I emphasize the word *rapidly* because you will find that the process is rather simple.

Along with teaching the skills, this module will also outline the real-world applications in which rapidly graphing a line is helpful to satisfy the demands of a business, commercial enterprise, financial planner, one's private endeavors, and so forth.

Before you draw anything, you must be familiar with two graphical concepts and their indispensable role in the construction of a line: I am referring to the *x-intercept* and the *y-intercept*. If these objects' definitions and significance are already very clear to you, then you may skip ahead. The next few boxes will nevertheless serve as a succinct refresher to these twin concepts if you are less than confident, or if you are lacking a thorough understanding of the material.



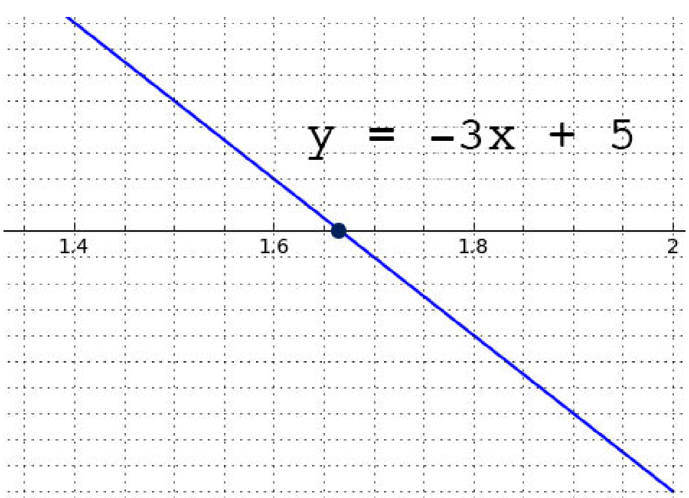
Definition of the X-Intercept

There are two ways you can understand the *x-intercept* of a linear equation:

Graphically: The point at which the line that represents the equation crosses the x-axis. Note, the x-axis is also the line $y = 0$.

Algebraically: The (x, y) pair that results when 0 is substituted for y in a linear equation and the equation is subsequently solved for x.

In the accompanying plot, it is apparent through inspection that the x-intercept for the equation $y = 4x - 2$ is $(0.5, 0)$. We can confirm this by plugging in 0 for y in the equation and seeing that we obtain the expression $x = 0.5$ after solving for x.



Let's try the following example now. Find the x-intercept of the line graphed by the equation $y = 5 - 3x$. Express your answer as a coordinate.

To solve, simply plug 0 for y in the equation, then solve for x.

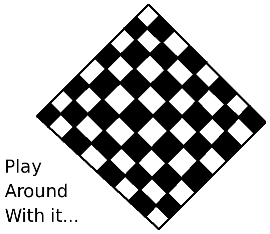
$$\begin{aligned} y &= -3x + 5 \\ 0 &= -3x + 5 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

Having found the value of x to be $\frac{5}{3}$, we know the x-intercept to be the coordinate $(\frac{5}{3}, 0)$.

Checking your work when finding an x-intercept is very easy: all you must do is plug the found value of x back into the original linear equation and confirm that y evaluates to 0. This is not a difficult problem, but let's do it just once here. Considering our solution from the previous box,

$$\begin{aligned} y &= 5-3x \\ y &= 5-3\left(\frac{5}{3}\right) \\ y &= 5-5 \\ y &= 0 \end{aligned}$$

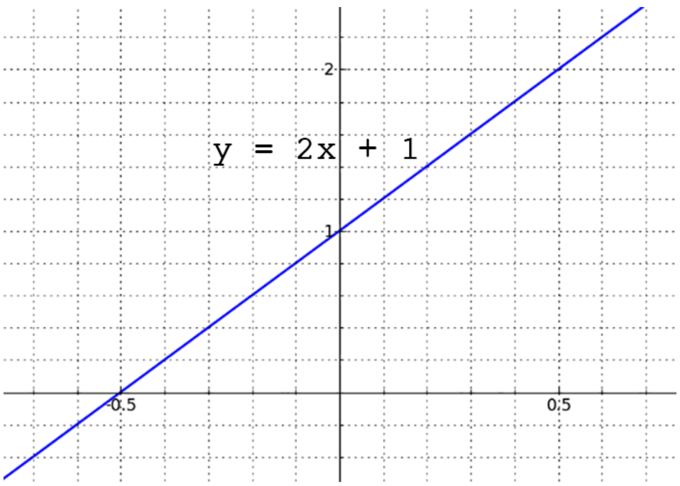
Note that the process of determining the x-value when finding the x-intercept is simply solving a one-variable linear equation. If you are comfortable performing these few steps, then it is very probable that your x-value will be correct and that checking your work might seem like overkill. However, since it only takes a few seconds, it never hurts to check your work.



1-3-1

Find the x-intercepts for the following linear equations. Note that each equation is expressed in slope-intercept form, $y = mx + b$:

- Find the x-intercept for $y = 4x + 10$. [Answer: $x = -2.5,\ y = 0$.]
- Find the x-intercept for $y = -\frac{15}{7}x - 20$. [Answer: $x = -\frac{28}{3},\ y = 0$.]
- Find the x-intercept for $y = -\sqrt{2}x + 8$. [Answer: $x = 5.65685\cdots,\ y = 0$.]



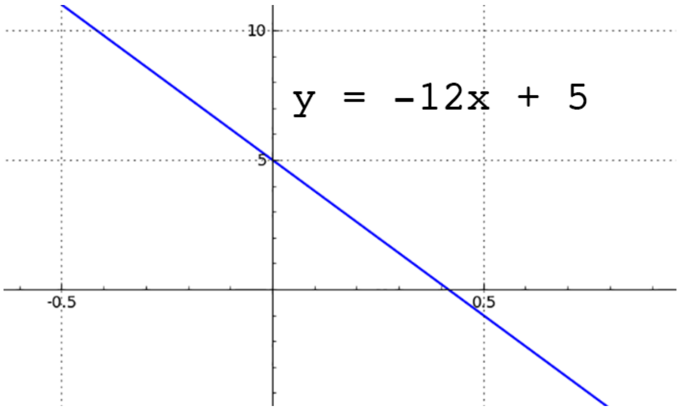
Definition of the Y-Intercept

There are two ways you can understand the *y-intercept* of a linear equation:

Graphically: The point at which the line that represents the equation crosses the y-axis. Note, the y-axis is also the line $x = 0$.

Algebraically: The (x, y) pair that results when 0 is substituted for x in a linear equation and the equation is subsequently solved for y.

In the accompanying plot, it is apparent through inspection that the y-intercept for the equation $y = 2x + 1$ is $(0, 1)$ or $x = 0,\ y = 1$. We can confirm this by plugging in 0 for x in the equation and seeing that we obtain the expression $y = 1$ after solving for y.



Find the y-intercept of the line graphed by the equation $y = -12x + 5$.

To solve, simply plug 0 for x in the equation, then solve for y. Express your answer as a coordinate.

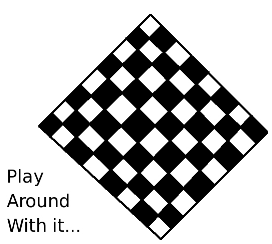
$$\begin{aligned} y &= -12x + 5 \\ y &= -12(0) + 5 \\ y &= 5 \end{aligned}$$

Having found the value of y to be 5, we know the y-intercept to be the coordinate $(0, 5)$.

Finding the y-intercept for a linear equation in slope-intercept form, $y = mx + b$, does not require any calculation whatsoever. Observe what happens when you substitute 0 for x, for any real number m:

$$\begin{aligned} y &= m(0) + b \\ y &= 0 + b \\ y &= b \end{aligned}$$

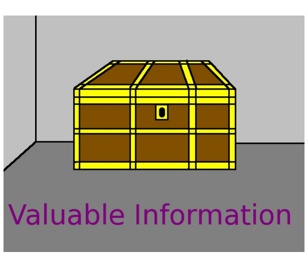
The coordinate of the y-intercept for lines in slope-intercept form is therefore always $(0, b)$. This can be quite convenient, and we'll explore it further in the next box.



1-3-2

Find the y-intercepts for the following linear equations. Observe that each equation is expressed in slope-intercept form, $y = mx + b$:

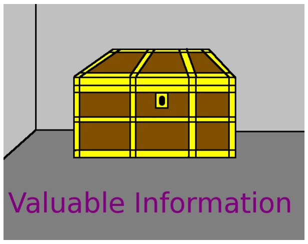
- Find the y-intercept for $y = 2.572x + \frac{17}{3}$. [Answer: $x = 0,\ y = \frac{17}{3}$.]
- Find the y-intercept for $f(x) = x - \sqrt{21}$. [Answer: $x = 0,\ f(x) = -\sqrt{21}$.]



For a linear equation in *any* form—not just slope-intercept form—the process of determining an intercept's location is the same: simply plug in 0 for one variable to find the intercept value of the other variable.

In fact, you will commonly see the following notation in the algebra work on math exams:

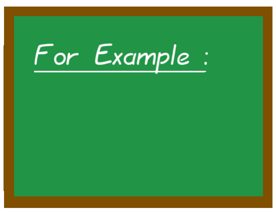
- if $x = 0$ then $y =$ ______________________________.
- if $y = 0$ then $x =$ ______________________________.



A line in *point-slope* form looks like

$$y - y_0 = m(x - x_0)$$

where y_0, x_0, and m are real numbers.



\# 1-3-3

Find the x- and y-intercepts of the following line in point-slope form:

$$y + 20 = 3(x - 12)$$

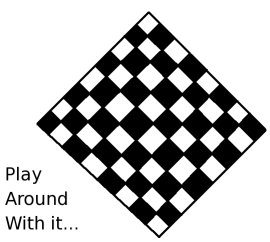
First we determine the x-intercept by plugging 0 for y into the equation:

$$\begin{aligned}(0) + 20 &= 3(x - 12)\\ 20 &= 3x - 36\\ 56 &= 3x\\ 18.6\overline{6} &= x\end{aligned}$$

indicating that the x-intercept occurs at the point $(18.6\overline{6},\ y = 0)$.
Similarly, to determine the y-intercept, substitute 0 for x in the equation:

$$\begin{aligned}y + 20 &= 3((0) - 12)\\ y + 20 &= 3(-12)\\ y + 20 &= -36\\ y &= -56\end{aligned}$$

indicating that the y-intercept is located at the coordinate $(0,\ -56)$.



\# 1-3-4

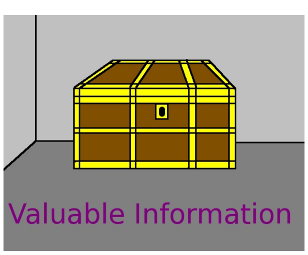
Determine the x- and y-intercepts of the following lines. Start each answer with the x-intercept, followed by the y-intercept.

- Find the intercepts for $y - 2.45 = 5.10(x + 7.35)$.

  [Answer: The intercepts occur at $(-7.83039\cdots, 0)$ and $(0, 39.935)$.]

- Find the intercepts for $y + 10.4 = 7(x - 3.8)$

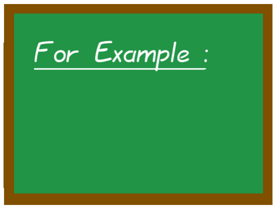
  [Answer: The intercepts occur at $(5.28571\cdots, 0)$ and $(0, -37)$.]



A line in *standard form* looks like

$$Ax + By = C$$

where A, B, and C are real numbers. Furthermore, it must not be the case that both of A and B are zero.



1-3-5

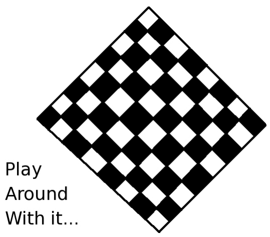
Determine the y-intercept for the following line in standard form:

$$4x - 5y = 11$$

The y-intercept is found by substituting 0 for x and then solving for y in the resulting single-variable linear equation.

$$\begin{aligned} 4(0) - 5y &= 11 \\ 0 - 5y &= 11 \\ -5y &= 11 \\ y &= -2.2 \end{aligned}$$

Now we can see that the y-intercept is at the coordinate $(0, -2.2)$.



1-3-6

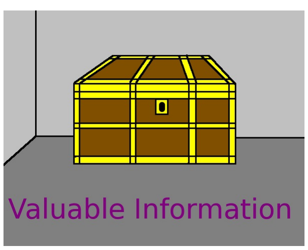
Determine the specified intercept for each of the following lines written in standard form.

- Find the y-intercept for $-y - 8x = 9$.

  [Answer: The y-intercept occurs at the coordinates $(0, -9)$.]

- Find the x-intercept for $\frac{14}{3}y + 5x = -\frac{24}{5}$.

  [Answer: The x-intercept occurs at the coordinates $(-0.96, 0)$.]

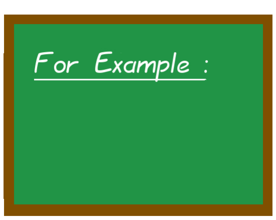


A line in *intercept-intercept* form looks like

$$\frac{x}{c} + \frac{y}{k} = 1$$

where c and k are non-zero real numbers.

As it comes to pass, it will always be the case that c is the x-intercept and k is the y-intercept. However, let's see an example or two, to confirm this fact.



1-3-7

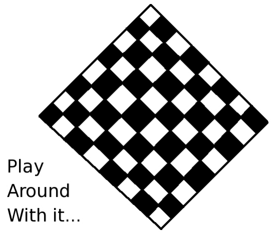
Determine the x-intercept for the following line in intercept-intercept form.

$$\frac{x}{6} + \frac{y}{12} = 1$$

Let's plug 0 in for y and solve for x.

$$\begin{aligned} \frac{x}{6} + \frac{0}{12} &= 1 \\ \frac{x}{6} &= 1 \\ x &= 6 \end{aligned}$$

Therefore the coordinate of the x-intercept is $(6, 0)$.



Find the x- and y-intercepts for the line

$$\frac{x}{8.7} - \frac{y}{10.3} = 1$$

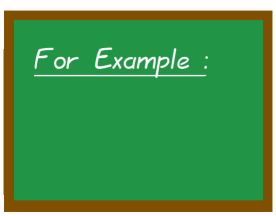
[Answer: The x-intercept is $(8.7, 0)$. The y-intercept is $(0, -10.3)$.]

1-3-8

The following is an extension of an example to be introduced on Page 295 of the module "Building a Linear Model."

Suppose a friend of yours sells ice cream from a cart in Central Park. He asks you to come up with an equation that predicts the number of cones sold in one day when the price is set at a particular level. Let x be the price of a cone and y be the number of cones sold in one day. Using a process that will be explained later, you devise the equation

$$y - 200 = -80(x - 2)$$



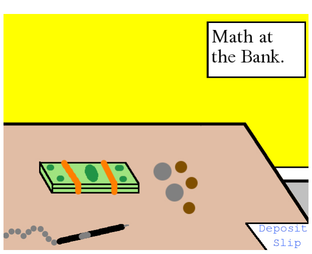
to relate price to the number of cones sold. At what price would your friend sell zero cones throughout the entire day?

This question is actually asking you to find an intercept of the line described by the above equation. Since the variable y represents the number of cones sold, you are interested in the case when $y = 0$, which is the x-intercept of the linear equation. Make this substitution in the equation and solve for x to determine the price at which zero cones are sold.

1-3-9

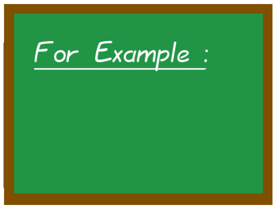
$$\begin{aligned} (0) - 200 &= -80(x - 2) \\ -200 &= -80x + 160 \\ -360 &= -80x \\ 4.5 &= x \end{aligned}$$

We will interpret this result in the next box.



In the previous box, we determined that if $x = 4.50$, then $y = 0$. This means that if a cone is priced at \$ 4.50, then no ice-cream cones are sold. In other words, the x-intercept occurs at $(4.5, 0)$. Later, in the "Supply and Demand" modules, we will learn that this is called the "maximum feasible price."

The type of equation that we are working with here is called a "price-demand" equation and it comes up very frequently in mathematical economics. Since the x-intercept had a meaning, you might wonder if the y-intercept also has a meaning. It does. We will see what that is, in the next box.



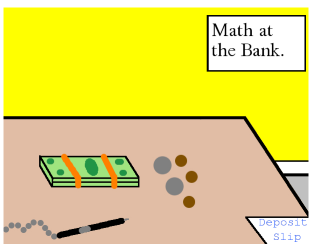
1-3-10

Looking at the previous example, suppose that your friend is using a batch of ice-cream mix whose expiration date is today. He would feel guilty if he charged his customers for almost-spoiled ice cream, so in an act of benevolence he decides to have a "Free Ice Cream Giveaway," since he plans on ordering a fresh batch of ice cream mix tomorrow. Based on the linear equation derived above, how many cones can your friend expect to "sell" if they are free (costing \$ 0)?

This question seeks to find the value of y (number of cones sold) when $x = 0$. (The price is zero.) The scenario asks you to identify the y-intercept of the linear equation, so we substitute 0 for x and solve for y.

$$\begin{aligned} y - 200 &= -80((0) - 2) \\ y - 200 &= -80(-2) \\ y &= 160 + 200 \\ y &= 360 \end{aligned}$$

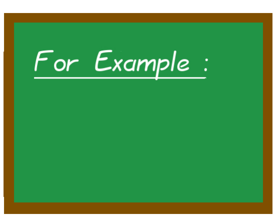
We will interpret this result in the next box.



In the previous box, we computed that if $x = 0$ then $y = 360$. This means that if cones are free for a day, then your friend can expect to give away 360 cones. In other words, the y-intercept occurs at $(0, 360)$. Later, in the "Supply and Demand" modules, we will learn that this is called the "saturation point."

In its fullest form, the theory of supply and demand is a phenomenally powerful technique in mathematical economics. It centers around a supply function and a demand function. Sometimes the supply function is hard to discover, but the demand function is much easier to find, and it turns out that you can do many things with the demand function alone. Of course, you probably don't know what a supply function or a demand function is yet, but that will be explained later, in the modules on "Supply and Demand."

The following is an extension of an example to be introduced on Page 305 of the module "Building a Linear Model."



1-3-11

Suppose a magazine is transitioning from the old paper media of the 20th century to online subscribers. The CEO is curious about the rate at which sales for the print edition of their publication are waning, so he asks an analyst to develop a linear equation (also known as a *linear model*) relating time to sales data. The analyst devises the following equation,

$$f_p(t) = -49.5t + 99{,}748$$

where t represents the year and $f_p(t)$ represents total sales of the print edition for that year. (You will derive this equation yourself when we get to Page 305.) Based on the linear model, when can the CEO expect sales of the print edition to reach zero?

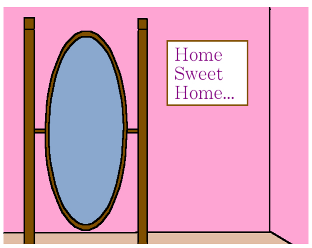
We'll answer this question in the next box. Note that although we chose t as the variable representing time rather than x, the function is still linear. The only difference is that we will have a t-axis instead of an x-axis.

Note, the p in $f_p(t)$ is decorative. It is there to tell us that this function is relating to print; there might be another $f(t)$ relating to online subscribers. The subscript p has no mathematical meaning here, but it is for human convenience. We will use subscripts very often throughout this book. We will continue this example in the next box.

The previous box asks you to find the t-intercept of the magazine's linear model: your goal is to discover the year in which print sales, represented by $f_p(t)$ in the equation, equal zero. Plug 0 in for $f_p(t)$ in the equation and solve for t to obtain your answer.

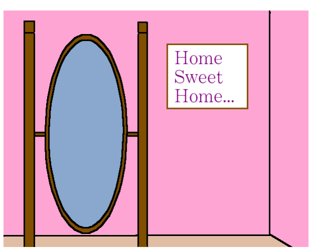
$$\begin{aligned} 0 &= f_p(t) \\ (0) &= -49.5t + 99,748 \\ 49.5t &= 99,748 \\ t &= 2015.1\overline{1} \end{aligned}$$

The t-intercept of the linear equation is $(2015.1\overline{1}, 0)$. In other words, the magazine is expected to sell no more print editions shortly after the start of 2015.



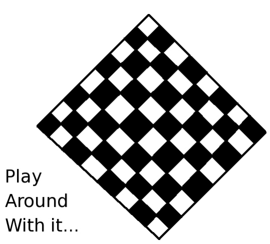
A Pause for Reflection...
You've surely noticed the fact that intercepts of a line are not just abstract algebraic objects. For businesses—large or small—whose production strategy depends upon keen insight into consumer behavior, the intercepts of an economic model provide key information about possible extreme trends within the market.

That people actually use the data revealed by intercepts to solve problems in the real world is an important secondary reason why this module teaches you the skill of finding them. Of course, we'll also use them for graphing, which is the primary reason.



A Pause for Reflection...
You noticed in the example about magazine sales that a variable t took the place of x in the linear equation. In this scenario, t was chosen because it represented *time*; we could have used any variable to represent time, but the convention in mathematics is to use t. Nevertheless, the model for the magazine business was still a two-variable linear equation; if we graphed it on a plane, the y-axis would be the vertical axis, and the t-axis would be the horizontal axis.

As you progress through this textbook, you will encounter many problems that describe a real-life scenario in which one quantity is varying over time. In these examples, don't be surprised if t is the variable representing time.



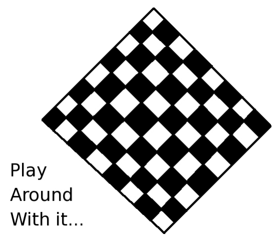
1-3-12

A local music venue has been in the concerts and promotions business for fifteen years, and they have developed a ticket pricing schedule which they have found accurately models ticket sales based on event pricing for unknown bands. The model

$$5y + 112.5x = 5625$$

relates the number of tickets sold for an event, y, to the price per ticket, x. Of course for a known band, especially a popular one, they might sell out completely, even with a relatively high price. If the venue wanted to promote the owner's son's band (a band which is not yet famous) by hosting a free concert one night, how many attendees should they expect?

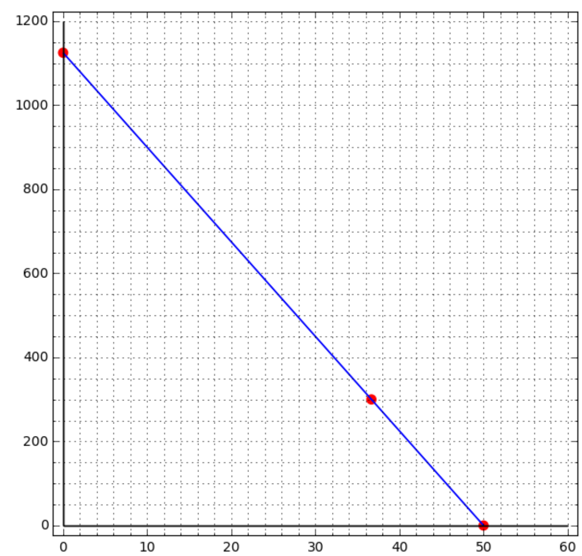
[Answer: If the ticket price were \$ 0, the venue should expect to "sell" (i.e. give away) 1125 tickets for the free show.]



1-3-13

Continuing with the previous box...

- Suppose that the annoying kid who lives across the street from the owner also has a band. That band wants to use this venue but the owner wants to "arrange things" so that only 300 tickets sell. What price would make this happen?
[Answer: $x = 36.6\overline{6}$, or \$ 36.66.]
- At what price would the venue expect to sell zero tickets? [Answer: $x = 50$ or \$ 50.]

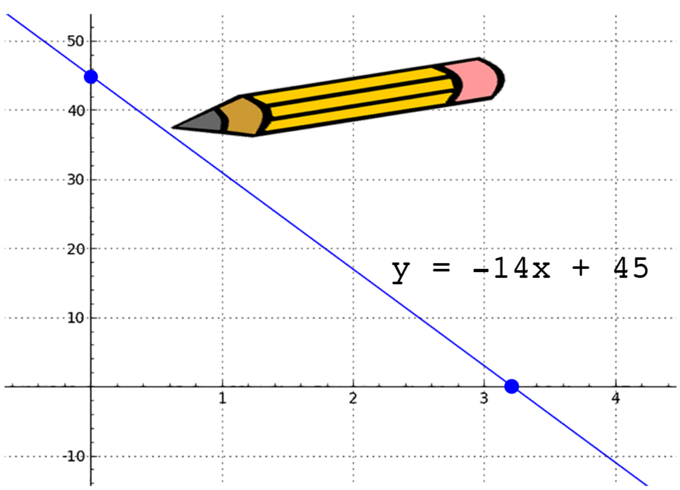


We did a fair amount of calculation to find the answers in the previous two boxes. We essentially had to solve an algebraic problem each time. Wouldn't it be nice to have a graph that could serve as a reference?

The graph is shown at the left of this box. The manager of the venue can simply look up the y-values and x-values visually.

For example, the three situations above are marked with three large dots. See if you can locate them.

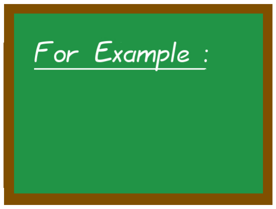
- $(0, 1125)$
- $(36.6\overline{6}, 300)$
- $(50, 0)$



Graphing a line is simply of matter of: first, finding two points that satisfy the linear equation—which we will refer to as the "anchor points" in this module; second, plotting those points on the coordinate plane; finally, third, drawing a straight line through those points. The only condition for the anchor points is that they must be distinct. Otherwise, *they can be any two points along the line.*

So why has this module been devoting so much time to finding the intercepts of a line?—because the intercepts are arguably the easiest points to find on a line. By substituting 0 for one variable, you greatly reduce the complexity of a linear equation in two variables, to a linear equation in one variable.

You should now get some practice graphing a line using its two intercepts, as this will be helpful when studying systems of inequalities later in this book. Once you have plotted two anchor points on a graph, just use a straightedge—like the edge of your textbook or a ruler—to trace a line connecting both points.



1-3-14

A car begins its descent from the summit of a 1400-meter mountain at $t = 0$. The altitude y of the car decreases steadily over time t (in minutes), and can be modeled by the linear equation

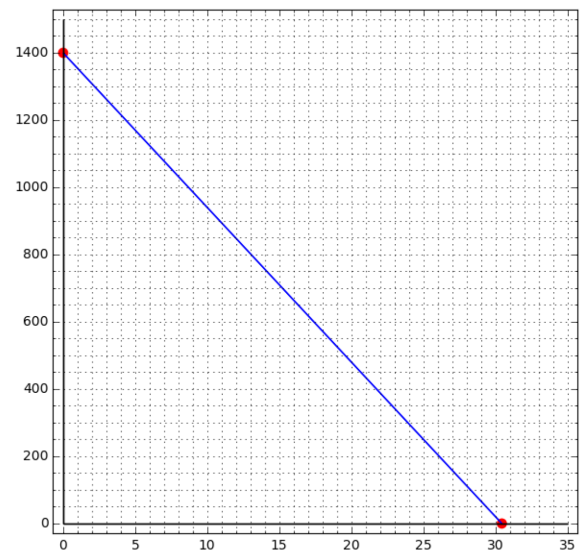
$$y = -46t + 1400$$

Find the t- and y-intercepts of this line and then graph the line, preferably on grid paper.

The first step in drawing the line is to find its t- and y-intercepts, which will become the anchor points of the line. Below, I will find the t-intercept in the left column and the y-intercept in the right column.

$$\begin{array}{rl}
y = -46t + 1400 & \quad y = -46t + 1400 \\
0 = -46t + 1400 & \quad y = -46(0) + 1400 \\
46t = 1400 & \quad y = 1400 \\
t = 30.4347\cdots & \\
-------- & \quad ------ \\
t\text{-int: } (30.4347\cdots, 0) & \quad y\text{-int: } (0, 1400)
\end{array}$$

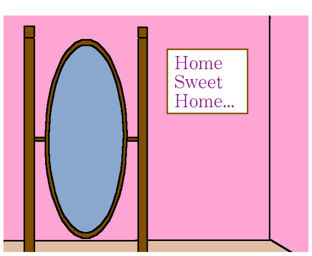
We will continue in the next box.



In the previous box we determined the t- and y-intercepts of the equation modeling a car's descent down a mountain. That's the first step. Second, I will draw those points on my graph paper.

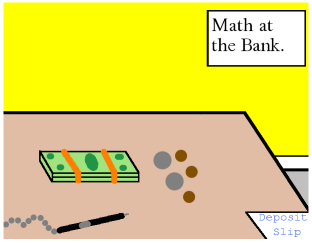
- $(0, 1400)$
- $(30.4347, 0)$

Third, to sketch this line, we plot the two intercepts on a coordinate plane (the t-axis should be the horizontal axis). Using a straightedge, we draw a straight line connecting and extending beyond the intercepts.



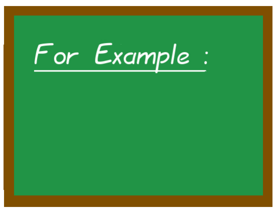
A Pause for Reflection...
You may be wondering if must we extend the line into the regions where $t < 0$ and $t > 30.4347\cdots$? Surely the car couldn't drive higher than the highest point of the mountain, nor will it relentlessly plow underground once it has reached the base of the mountain. Suffice it to say that although we recognize the model loses credibility beyond the two extreme values of $t = 0$ and $t = 30.4347\cdots$ (a mathematician would say the model "breaks down" past these points), we could choose to ignore this betrayal of reality and seek to graph the line for whatever t-values and y-values we want. Of course, on a quiz, test, or exam, you should draw the graph where you are told to draw it.

In future exercises throughout this module, do not hesitate to continue drawing a line out past the points that "make sense," since by definition a line has no beginning nor end, but rather extends infinitely in both directions.



In contrast to the previous box, we typically only show the values that make sense when graphing, especially in a textbook, or any work relating to science, business, or finance. Often the points that make sense are those with positive coordinates. This will remain true if you take advanced courses in mathematical economics or finance.

I should mention that pure mathematicians call the part of the coordinate plane with positive coordinates by the name "Quadrant 1" but that will not be an important vocabulary term for us in this textbook.



\# 1-3-15

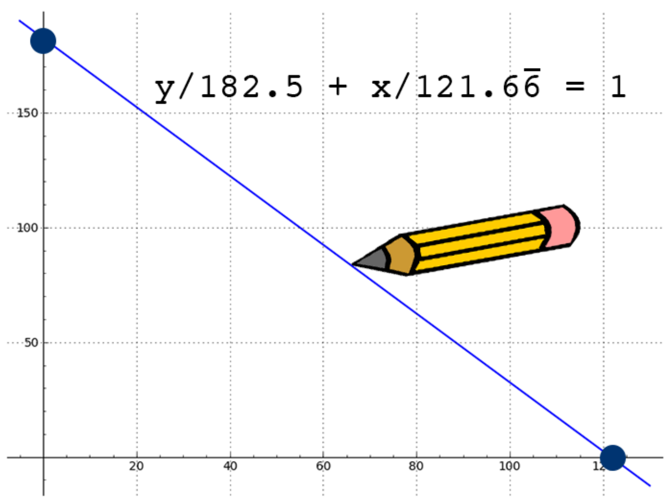
Let's practice graphing another line together before you attempt a few exercises on your own.

A premium cable package estimates that its monthly subscription numbers y correlate directly with the price x of a subscription. The consumer demand of a cable subscription is modeled as follows:

$$\frac{x}{121.6\overline{6}} + \frac{y}{182.5} = 1$$

Sketch the line that expresses this relation between price x and subscriptions y.

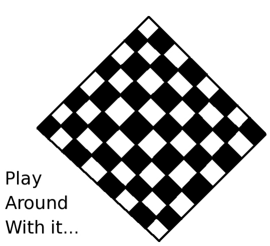
We'll solve this in the next box.



Continuing with the previous box, let's start off by finding the intercepts of the linear equation. Below, in the left column we shall determine the x-intercept and in the right column we will determine the y-intercept.

$$\begin{array}{cc}
\dfrac{x}{121.6\overline{6}} + \dfrac{(0)}{182.5} = 1 & \dfrac{(0)}{121.6\overline{6}} + \dfrac{y}{182.5} = 1 \\
\dfrac{x}{121.6\overline{6}} = 1 & \dfrac{y}{182.5} = 1 \\
x = 121.6\overline{6} & y = 182.5 \\
------- & ------- \\
x\text{-int: } (121.6\overline{6}, 0) & y\text{-int: } (0, 182.5)
\end{array}$$

By plotting the two intercepts on a coordinate plane and connecting them with a straight line, you will have successfully graphed the linear equation.



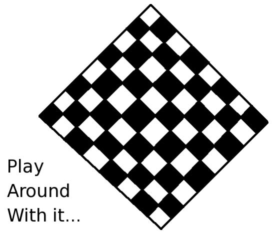
\# 1-3-16

Graphing Exercise 1

Find the two intercepts for the following line, then graph that line (preferably on graph paper, if you have it).

$$110y \quad = \quad 680 - 42x$$

The answer to this exercise can be found at the end of this module, on Page 134, as Graphing Exercise # 1.



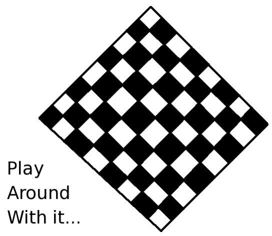
1-3-17

Graphing Exercise 2

Find the two intercepts for the following line, then graph that line (preferably on graph paper, if you have it).

$$-4.6x + 1.8y \quad = \quad -16$$

The answer to this exercise can be found at the end of this module, on Page 135, as Graphing Exercise # 2.



1-3-18

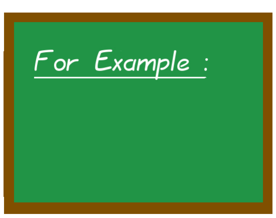
Graphing Exercise 3

Find the two intercepts for the following line, then graph that line (preferably on graph paper, if you have it).

$$9y - 6 \quad = \quad 5(6x + 15)$$

The answer to this exercise can be found at the end of this module, on Page 135, as Graphing Exercise # 3.

So far, the only method for graphing lines this module has exposed you to is by using the x-and y-intercepts as anchor points. There are three cases, however, where this method is either insufficient or impractical. I will now illustrate these three scenarios and explain how to resolve their particular complications.



1-3-19

A particular lightbulb filament has been recalled by the manufacturer because it has been demonstrated to steadily increase heat output over time, eventually causing the lightbulb to combust. The rate of its heat increase $f(t)$ (in thousands of degrees Celsius) over time t (in hours) is modeled by the linear equation

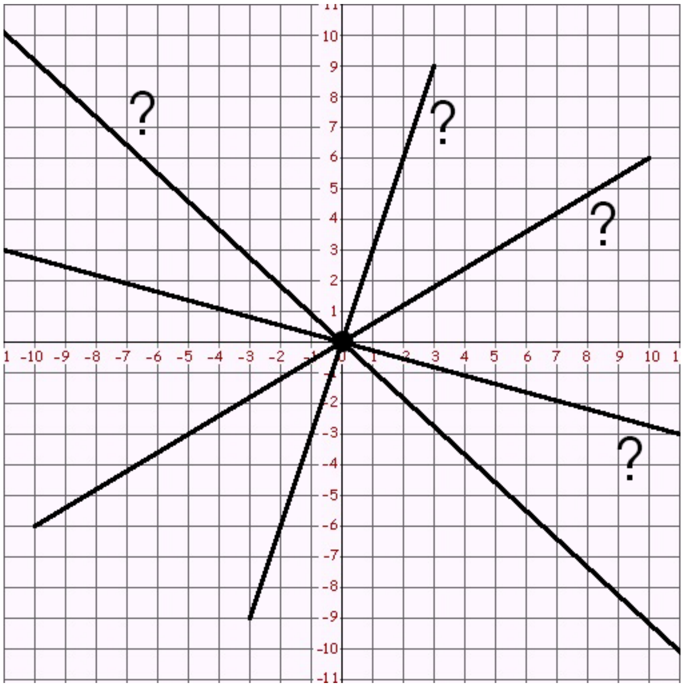
$$f(t) = 0.75t$$

Graph this line onto a coordinate plane after finding its t- and $f(t)$- intercepts.

Below, in the left column we will find the t-intercept, and in the right column we will find the $f(t)$-intercept.

$$\begin{array}{rl}
f(t) = 0.75t & \quad f(t) = 0.75t \\
(0) = 0.75t & \quad f(t) = 0.75(0) \\
0 = 0.75t & \quad f(t) = 0 \\
0 = t & \\
--- & \quad --- \\
(0,0) & \quad (0,0)
\end{array}$$

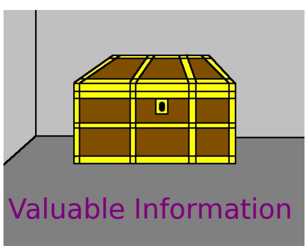
What happened here? It is evident that the t- and $f(t)$-intercepts are the same point, $(0,0)$. This point is so important that it has a name—it is called *the origin*. In the next box we will discuss why we cannot rapidly graph this line just yet, as we have been doing so far. We will generalize this situation in the next box.



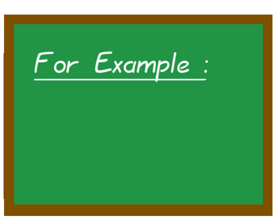
Problematic Scenario: The Line Cuts Through the Origin

In the equation $f(t) = 0.75t$ the two intercepts turn out to be the same point on the coordinate plane, the origin $(0, 0)$. Our method for finding adequate anchor points for a line has failed us, since one cannot plot a line with only one anchor point.

Suppose you tried to graph a line with only one anchor point. Although you have found a fixed point through which the line travels, how are you supposed to know in what direction it travels? It is simply impossible to tell, until you find a second anchor point, distinct from the origin.



When you encounter a line that intersects the origin of the coordinate plane—that is, point $(0, 0)$—you must find a second distinct point on that line to use as an anchor point when graphing the line on paper.



1-3-20

We are trying to figure out how to graph the equation

$$f(t) = 0.75t$$

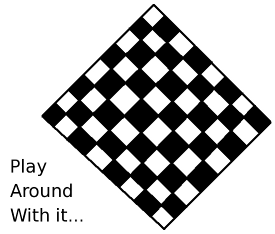
which is challenging because the intercepts only provide us with one anchor point, $(0, 0)$.

All we have to do is find any other point on the line. We can plug in any value other than 0 for t that we like, and we obtain a non-zero value for $f(t)$, giving us a new point. Let's try $t = 4$, because that will cancel nicely with $0.75 = 3/4$. We obtain

$$\begin{aligned} f(t) &= 0.75t \\ f(t) &= 0.75(4) \\ f(t) &= 3 \end{aligned}$$

Now we know that $(4, 3)$ is a point on the line, as well as $(0, 0)$. With two anchor points, we can now draw the line.

We could have chosen any value of t to find our second point, but $t = 4$ was a convenient value.



\# 1-3-21

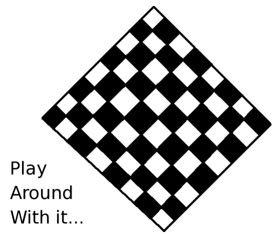
Graphing Exercise 4

The following equation intersects the origin:

$$4f(x) = -10x$$

Find a second point that lies on the line, apart from the origin, and graph the line.

See Page 135 for a graph of the line, as Graphing Exercise # 4.



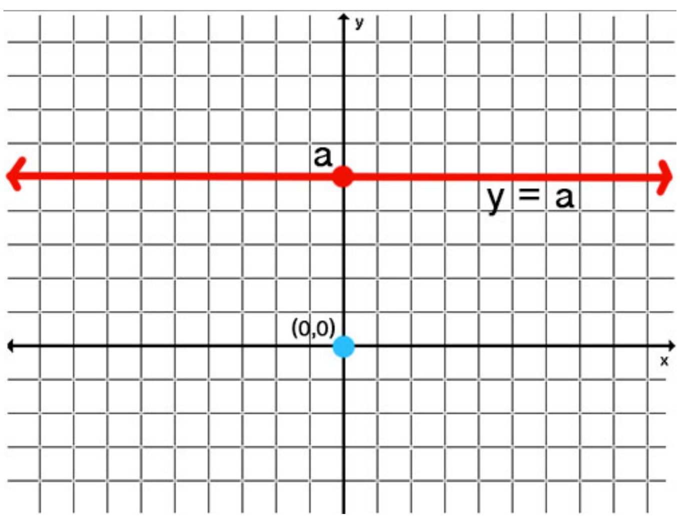
\# 1-3-22

Graphing Exercise 5

Simplify the following equation and graph it on a coordinate plane:

$$2y + 6 = 2(0.8x + 3)$$

See Page 136 for a graph of the line, as Graphing Exercise # 5.

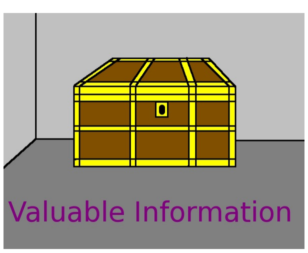


Scenario 2: Line Only Intersects Y-Axis (Horizontal Line)

Assume that after simplifying a linear equation, you obtain the equation:

$$y = a$$

where a is any real number. What you have here is a line whose y-value is not dependent upon its x-value. In other words, the line has a consistent y-value across the entire coordinate plane—it is a perfectly horizontal line.

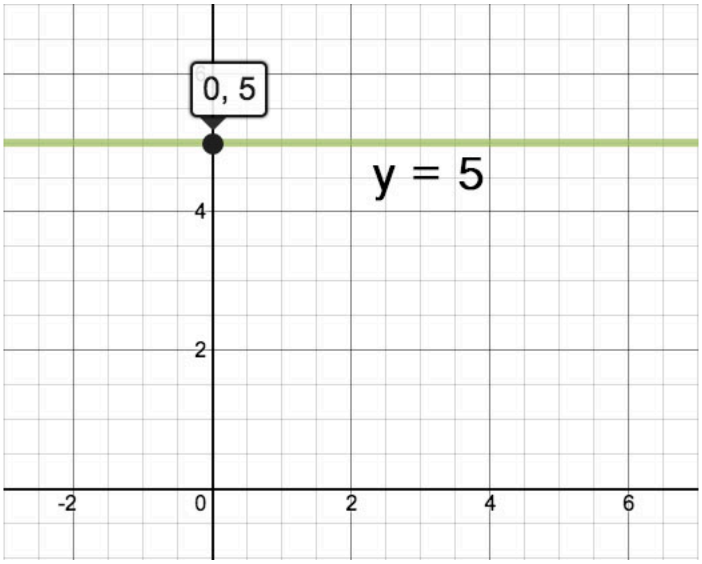


When you have a linear equation that resembles

$$y = a$$

where a is any real number, then the line is a horizontal line intersecting the y-axis at $y = a$.

To graph this equation, simply draw a line cutting through the point $(0, a)$ that is perpendicular to the y-axis and parallel to the x-axis.

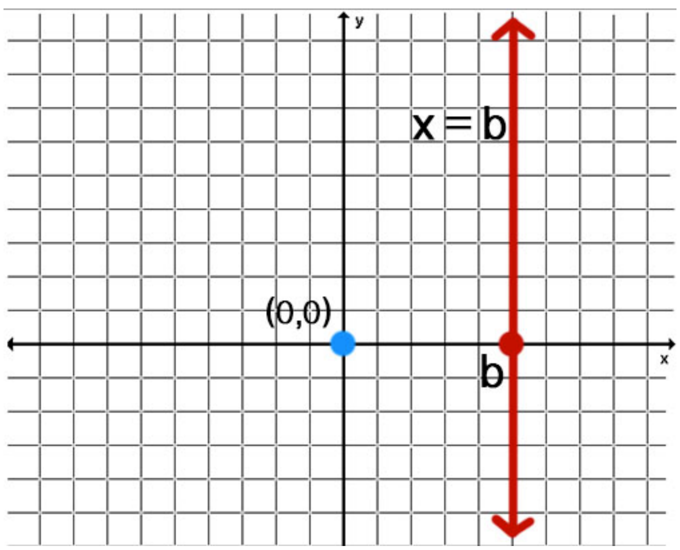


Graphing a Horizontal Line

Assume you are asked to graph a linear equation which simplifies to $y = 5$.

First, you observe that there is no instance of x in the equation. This indicates that the y-coordinate of any individual point on the line is not dependent on its x-coordinate. Rather, the entire graph of the equation lies along the gridline $y = 5$, seen in the graph to the left.

To rapidly graph a horizontal line on the coordinate plane, find the y-value at which the line intersects the y-axis, and using a straightedge trace a path with your pencil that runs parallel to the x-axis. Remember that a horizontal line lying above or below the x-axis will never intersect the x-axis.

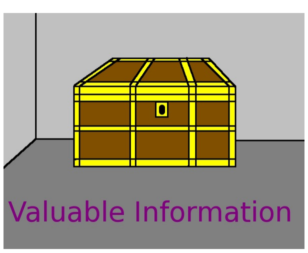


Scenario 3: Line Only Intersects X-Axis (Vertical Line)

Assume that after simplifying a linear equation, you obtain the relation

$$x = b$$

where b is any real number. What you have here is a line whose x-value never changes for any y-value. In other words, the line has a consistent x-value across the entire coordinate plane—it is a perfectly vertical line.

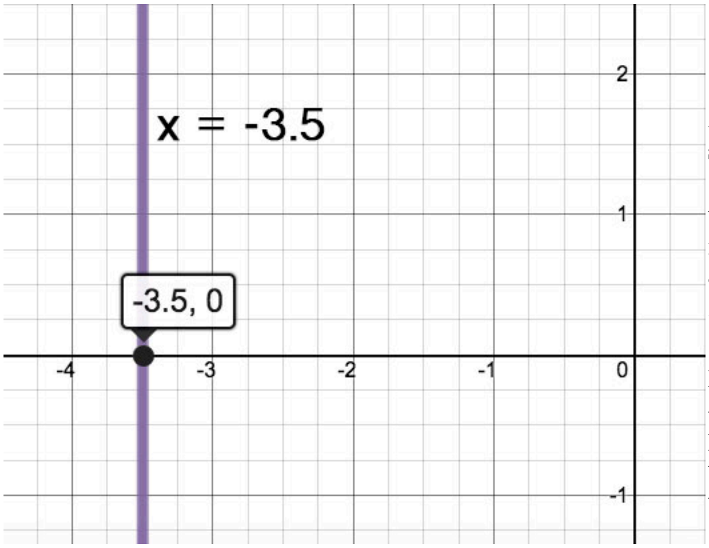


When you have a linear equation that resembles

$$x = b$$

where b is any real number, then the line is a vertical line intersecting the x-axis at $x = b$.

To graph this line, simply draw a line cutting through the point $(b, 0)$ that is perpendicular to the x-axis and parallel to the y-axis.



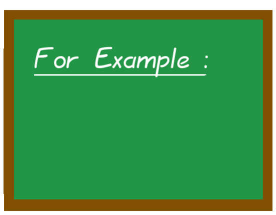
Graphing a Vertical Line

Assume you are asked to graph a linear equation which simplifies to $x = -3.5$.

First, you observe that there is no instance of y in the equation. This indicates that the x-coordinate of any individual point on the line is not dependent on its y-coordinate. Rather, the entire graph of the equation lies along the gridline $x = -3.5$, seen in the graph to the left.

To rapidly graph a vertical line on the coordinate plane, find the x-value at which the line intersects the x-axis, and using a straightedge trace a path with your pencil that runs parallel to the y-axis. Remember that a vertical line lying to the left or to the right of the y-axis will never intersect the y-axis.

We have now completed our review of how to graph lines. As a capstone exercise, we will solve the two problems relating to graphing that appeared in Diagnostic Exercise Two, on Page 87. The only thing that is different is that we are going to draw two lines on the same graph.



1-3-23

During the algebra diagnostic, Diagnostic Exercise Two, you were asked to carefully graph the following lines:

$$5x + 4y = 3 + 4x + 3y$$

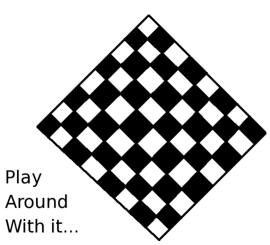
as well as

$$2x - 2y = 8 - 3y$$

over the interval $-2 \leq x \leq 10$, and the interval $-12 \leq y \leq 12$.

By the way, when we say "over the interval $-2 \leq x \leq 10$," that signifies the values of the x-axis which should be visible on your graph. Likewise, "over the interval $-12 \leq y \leq 12$," signifies the values of the y-axis that should be visible. This is similar to setting the window on a graphing calculator. You are choosing what ranges of xs and ys should be visible.

In any case, you will now complete the graph yourself, in the next box.

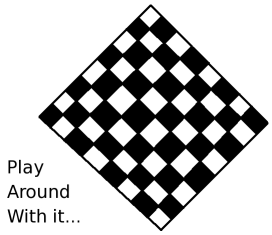


1-3-24

Consider the two equations given in the previous box.

- What are the coordinates of the x-intercept of the first equation? [Answer: $(3, 0)$.]
- What are the coordinates of the y-intercept of the first equation? [Answer: $(0, 3)$.]
- What are the coordinates of the x-intercept of the second equation? [Answer: $(4, 0)$.]
- What are the coordinates of the y-intercept of the second equation? [Answer: $(0, 8)$.]
- Now, draw the graph of these two equations, on the graph.

You can find the correct graph on Page 136, as Graphing Exercise # 6.



1-3-25

Please carefully graph the following lines, on the same graph.

$$7x + 4y = 10 + 2x + 2y$$

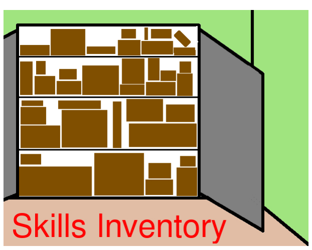
as well as

$$8x + 5y = 18 + 2x + 2y$$

over the interval $-4 \leq x \leq 6$, and the interval $-10 \leq y \leq 15$.

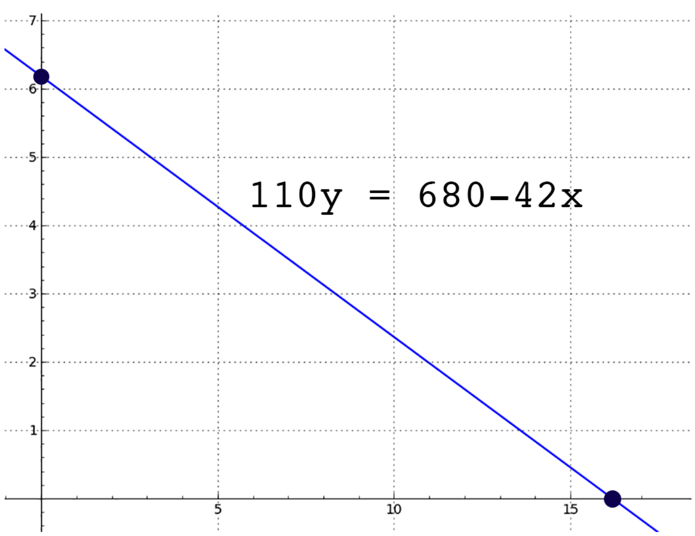
- What are the coordinates of the x-intercept of the first equation? [Answer: $(2, 0)$.]
- What are the coordinates of the y-intercept of the first equation? [Answer: $(0, 5)$.]
- What are the coordinates of the x-intercept of the second equation? [Answer: $(3, 0)$.]
- What are the coordinates of the y-intercept of the second equation? [Answer: $(0, 6)$.]
- Now, draw the graph.

You can find the correct graph on Page 136, as Graphing Exercise # 7.



In this module we learned or reviewed:

- How to find the intercepts of a line, given its equation in slope-intercept form, point slope form, general form, or intercept-intercept form.
- How to use the intercepts of a line as the two anchor points when graphing the line.
- How to produce a new anchor point if both intercepts turn out to be the origin.
- The form of an equation representing a perfectly horizontal line or vertical line.
- How to graph a system of linear equations using the above principles for each equation in the system.
- …and the vocabulary terms "the origin," x-intercept, and y-intercept.

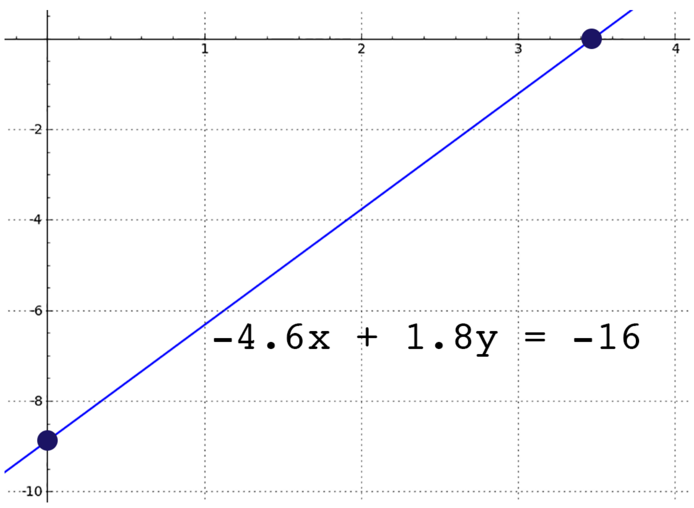


Graphing Exercise 1

To the left is the plot corresponding to the checkerboard box on Page 128 of this module.

The x-intercept of this graph is $(16.1904\cdots, 0)$.

The y-intercept of this graph is $(0, 6.18181\cdots)$.

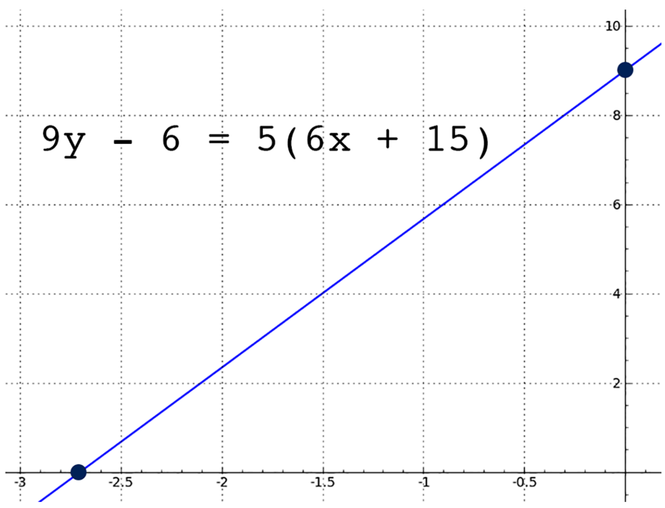


Graphing Exercise 2

To the left is the plot corresponding to the checkerboard box on Page 129 of this module.

The x-intercept of this graph is $(3.47826\cdots,0)$.

The y-intercept of this graph is $(0,-8.8\overline{8})$.

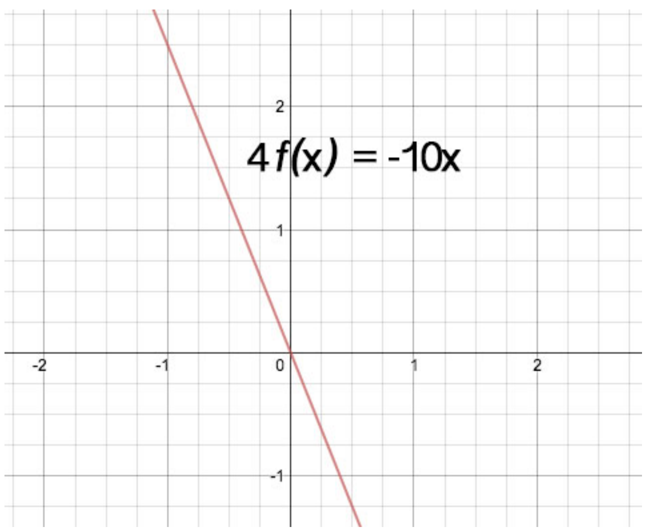


Graphing Exercise 3

To the left is the plot corresponding to the checkerboard box on Page 129 of this module.

The x-intercept of this graph is $(-2.7,0)$.

The y-intercept of this graph is $(0,9)$.

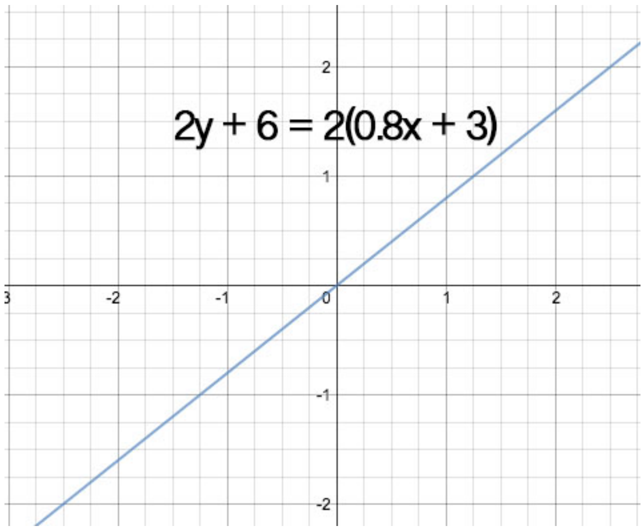


Graphing Exercise 4

To the left is the plot corresponding to the checkerboard box on Page 131 of this module.

When simplified, the linear equation $4f(x) = -10x$ becomes $f(x) = -2.5x$.

This graph of this linear equation intersects the origin $(0,0)$.

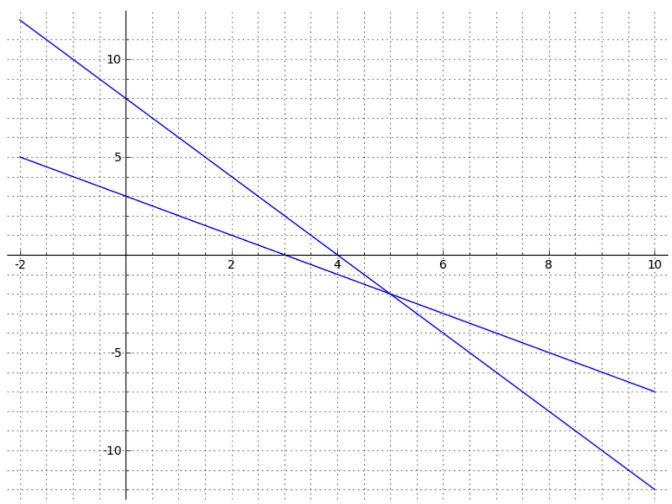


Graphing Exercise 5

To the left is the plot corresponding to the checkerboard box on Page 131 of this module.

When simplified, the linear equation $2y + 6 = 2(0.8x + 3)$ becomes $y = 0.8x$.

This graph of this linear equation intersects the origin $(0, 0)$.



Graphing Exercise 6

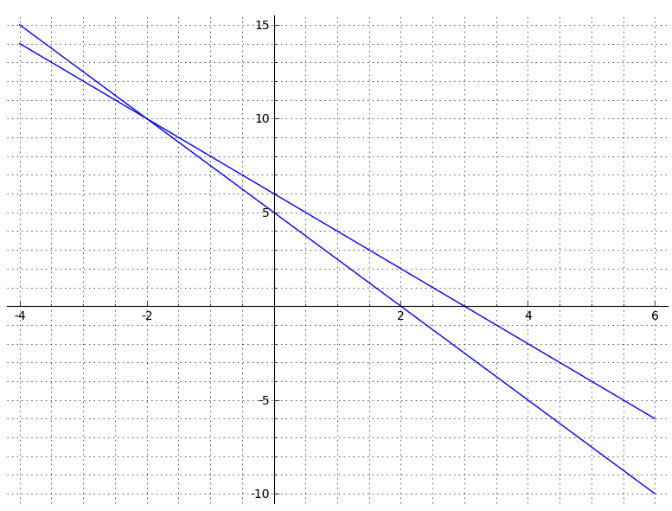
Here is the graph for Exercise # 6.

$$5x + 4y = 3 + 4x + 3y$$

as well as

$$2x - 2y = 8 - 3y$$

The steeper line is the second equation, and the shallower line the first equation, if you are curious.



Graphing Exercise 7

Here is the solution to Exercise # 7.

$$7x + 4y = 10 + 2x + 2y$$

as well as

$$8x + 5y = 18 + 2x + 2y$$

This time, the steeper line is the first equation, and the shallower line the second equation, if you are curious.