Improvements that Pay for Themselves

Math-123: Finite & Financial Mathematics

February 13, 2017

Questions Only

1. In a real-estate firm, where some nice color brochures are made for each house sold, the manager is concerned about the expenditures on printer ink. The old printer costs $ 2 per page for color graphics. Replacing it will cost $ 600, but the new printer will cost $ 0.75 per page. How many pages need to be printed before the new printer pays for itself?

    Let p be the number of printed pages.

2. A particular restaurant has one wall that is almost entirely windows, with a nice view of the lake. The heating and the air-conditioning both struggle with this, because the windows leak/trap heat. Replacing the windows with double-glazed windows will cut the monthly electric bill from $ 900 to $ 400. However, the installation will cost $ 20,000. How many months will it take for the new windows to pay for themselves?

    Let m be the number of months.

3. For a major public relations firm, taking clients to and from the airport costs $ 100 per one-way trip, because they use a nice sedan service. Buying one of those shinny new luxury sedans would cost around $ 42,000, but then any employee could be the driver. The only cost would be the gas, and hourly wages for the employee, coming to a total of perhaps $ 40. How many trips would it take for the sedan to pay for itself?

    Let n be the number of one-way trips.

4. A software-development firm has long had the tradition of providing free coffee for its staff. They typically go through 6 scoops per day of cheap coffee for their drip-style coffee maker, and a can contains 80 scoops, costing $ 38.00. Instead, they can buy a nice coffee maker that uses a pod for each cup. They anticipate 36 cups per day will be used, and each pod costs $ 0.50. The new coffee maker costs $ 190. How long will it take for the pod-based coffee maker to pay for itself?

    Let d be the number of days.

5. A struggling firm that makes advertising for the web is thinking of downsizing their offices. Currently, they have offices in Manhattan, and pay $ 6000 per month. They can move to Brooklyn, and pay $ 3000 per month. However, it will require $ 12,000 of mover's fees, plus a $ 6000 deposit on the new offices. How long will it take the move to pay for itself?

    Let m be the number of months.

Questions & Answers

1. In a real-estate firm, where some nice color brochures are made for each house sold, the manager is concerned about the expenditures on printer ink. The old printer costs $ 2 per page for color graphics. Replacing it will cost $ 600, but the new printer will cost $ 0.75 per page. How many pages need to be printed before the new printer pays for itself?

   Let p be the number of printed pages.

   $$\begin{aligned} 2p &> 600 + 0.75p \\ 1.25p &> 600 \\ p &> 600/1.25 \\ p &> 480 \end{aligned}$$

   Therefore, it will take 480 pages before the new printer pays for itself. (If they print 4 pages per week, then this is not attractive. If they print 100 pages per week, then this is very attractive.)

2. A particular restaurant has one wall that is almost entirely windows, with a nice view of the lake. The heating and the air-conditioning both struggle with this, because the windows leak/trap heat. Replacing the windows with double-glazed windows will cut the monthly electric bill from $ 900 to $ 400. However, the installation will cost $ 20,000. How many months will it take for the new windows to pay for themselves?

   Let m be the number of months.

   $$\begin{aligned} 900m &> 20,000 + 400m \\ 500m &> 20,000 \\ m &> 20,000/500 \\ m &> 40 \end{aligned}$$

   It will take 40 months, or 3 years and 4 months, for the new windows to pay for themselves. This sounds attractive, if the restaurant has the capital on hand. However, if they are short of capital, then they would probably decline the upgrade. If they get a loan for the upgrade, then a more careful calculation must be made, including the interest and other finance charges.

3. For a major public relations firm, taking clients to and from the airport costs $ 100 per one-way trip, because they use a nice sedan service. Buying one of those shinny new luxury sedans would cost around $ 42,000, but then any employee could be the driver. The only cost would be the gas, and hourly wages for the employee, coming to a total of perhaps $ 40. How many trips would it take for the sedan to pay for itself?

   Let n be the number of one-way trips.

   $$\begin{aligned} 100n &> 42,000 + 40n \\ 60n &> 42,000 \\ n &> 42,000/60 \\ n &> 700 \end{aligned}$$

   As you can see, it will require 700 trips before the sedan pays for itself. One really has to stop and think.... If there's only one trip per week, that's 13.4615 years. Even if there's actually two trips per

week, that's 6.73076 years. The sedan will have aged a lot by then. Of course, if there's one trip per day, then the sedan will pay for itself before the end of two years.

4. A software-development firm has long had the tradition of providing free coffee for its staff. They typically go through 6 scoops per day of cheap coffee for their drip-style coffee maker, and a can contains 80 scoops, costing \$ 38.00. Instead, they can buy a nice coffee maker that uses a pod for each cup. They anticipate 36 cups per day will be used, and each pod costs \$ 0.50. The new coffee maker costs \$ 190. How long will it take for the pod-based coffee maker to pay for itself?

Let d be the number of days.

- With the old coffee maker, in d days, we use $6d$ scoops. That's $6d/80$ cans or $(6d/80)(38)$ dollars.
- With the new coffee maker, in d days, we use $36d$ pods, for a cost of $36d(0.50)$ dollars. (Plus \$ 190 for the machine itself.)

$$
\begin{aligned}
\left(\frac{6d}{80}\right)38 &> 190 + (36d)(0.50) \\
2.85d &> 190 + 18d \\
-15.15d &> 190 \\
d &< 190/(-15.15) \\
d &< -12.5412\cdots
\end{aligned}
$$

This says that it will take a negative number of days for the pod-based coffee maker to pay for itself. You cannot have a negative number of days, and that means the pod-based coffee maker will never pay for itself. It is, in fact, a strictly losing proposition.

This can be seen by realizing that the pods cost $36d(0.50) = 18d$, or \$ 18 per day. The drip-style coffee maker costs $(6d/80)(38)$ or \$ 2.85 per day.

5. A struggling firm that makes advertising for the web is thinking of downsizing their offices. Currently, they have offices in Manhattan, and pay \$ 6000 per month. They can move to Brooklyn, and pay \$ 3000 per month. However, it will require \$ 12,000 of mover's fees, plus a \$ 6000 deposit on the new offices. How long will it take the move to pay for itself?

Let m be the number of months.

$$
\begin{aligned}
6000m &> 3000m + 12,000 + 6,000 \\
6000m &> 3000m + 18,000 \\
3000m &> 18,000 \\
m &> 18,000/3000 \\
m &> 6
\end{aligned}
$$

As you can see, in only six months, the move will have paid for itself. However, that means that the cash-flow benefits of the cheaper offices will only be felt starting in the seventh month. Will the firm last that long? Who knows!