Some Challenging Word Problems

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Questions

1. You are the head of a sales division consisting of three sales teams selling cars. Team A sold 1 fewer car than Team B, and Team B sold 3 more than Team C. If the division sold 116 cars total, then how many cars did each team sell individually?

2. Two factories in China are for sale, and their total market value is 900 million dollars. If one factory is worth triple the cost of the other, then what is the fair market value of each?

3. Alice and Bob compare bank balances, and Alice teases Bob that if she had only $ 3 more, she'd have exactly triple what Bob has. Together they total $ 1568.48. How much does each have?

Note: To show the diversity of (entirely correct) answers I get back from students, I have written up three different ways to solve the previous problem in the answers section. Two of the ways only involve equations in one variable.

4. A charity is conducting a telethon to raise money for the vaccination of children in sub-Saharan Africa. During one of the breaks the charity's director states that they are $ 5900 short of a quarter of their goal. The announcer asks how much they actually have, but the director elusively replies that they have one sixth of their goal. Clearly the director of the charity is trying to hide that dollar value, but he has failed to hide it, because the information he has already revealed actually makes the goal easy to calculate. What is the dollar value of the charity's goal, and how many dollars do they have so far?

5. The safety officer of a factory with 42 accidents last year notes that the milling machine was responsible for 8 more accidents than all of the other machines combined. How many accidents occurred on the milling machine?

6. There are four poker players seated at a table. Only one walks away with gains. Of the winner's pot, 20% came from Alvin, and Simon lost $ 100

more than Alvin did. Meanwhile Theodore lost $ 60 more than Simon. How much did each loser lose? How much did the winner walk away with?

7. Let us suppose your former boss has a summer home near the beach in Florida, and her job moves her to Europe for a few years. She does not want to sell the summer home, so naturally she rents it out to get some income. As you can imagine, the rental price is higher in May, June, July and August than in the rest of the year. She sees you at a conference, and tells you that in the year 2010, fire damage caused her to be unable to collect rent for August or any later month of that year. However, she still got $ 9,105 for the year. You recall from a previous conversation that she anticipates $ 15,060 in a typical year. What rent does she charge during the summer months? During the off-season months?

8. A hobbyist has tried making wine at home using a kit, and the results are rather good. A buyer that he knows offers him $ 8 per bottle. He thinks he can get at least ten dollars per bottle, so he tells the buyer to return a different day. In the meantime, the buyer loses interest, and the winemaker gives away two bottles to his mother and mother-in-law. However, he later gets another offer for $ 9.50 per bottle and accepts, this time selling all the bottles he now has left. His mother-in-law reprimands him for wasting two bottles as gifts, but he points out that he made $ 23 more on the latter sale than he would have made had he accepted the prior offer. How many bottles were there in the final sale?

9. You are a factory manager, and a union boss is coming up for re-election. Most workers favor the current union boss. In fact, he's up by 400 votes versus his opponent. However, employees are not allowed to vote unless they've been working there for more than one year. The 200 junior employees are split 50-50 in their opinion.

   You'd like to see the union boss changed. Your foreman warns you that even if you allow the junior employees to vote, the union boss would still win his re-election by about 2:1. How many people will vote which way if you do not allow the junior employees to vote?

Removed Problems: These last few problems were at one time removed because they are not taking place in a business-related setting. However, if you need more practice, then feel free to use them.

10. Two rare Ancient Greek coins are for sale, together for $ 880. If one is worth triple the cost of the other, what is the fair market value of each?

11. A World War II infantry commander has a mixed Canadian-British unit. He has twice as many Brits as Canadians. Then he loses 700 of each. Now he has three times as many Brits as Canadians. How many did he start with of each?

12. An acrimonious debate is going on at the student council of a local university, where juniors and seniors are permitted to park on campus, but not freshmen and sophomores, who must park in an off-campus lot. The council is considering filing a petition to request that sophomores have the right to park on campus. During the debate, one person claims that "the majority of sophomores have cars," but he is promptly contradicted. The campus-parking office representative rebuts that there are only 1437 sophomores who have a registered vehicle, which is 29 short of half the number of sophomores at the university. Later that evening, while preparing the petition, it would be convenient to know the number of sophomore students at the university, but that information does not appear to be on the university's website. Infer the number of sophomores from the information given here.

13. There has been an accident in a lab of a space station, and some toxic gases have been released into the lab module. There is a pump from the vessel's oxygen supply, into the room, and a pump from the room into the vessel's exhaust system. First, the commander pumps all the air in the room out, using the exhaust pump. Second, once this is complete, the commander turns on the oxygen pump, with the exhaust pump not running, and the room fills with oxygen. Third, she turns on both pumps and the module is airless after 12 hours go by. Fourth, she turns on the oxygen pump for 3 hours. Fifth, she leaves the oxygen pump running, and turns on the exhaust pump again, and the module is empty after 9 hours. The commander now feels that the toxic gases have been cleaned out, but she has to make entries in the ship's log about the accident. How long did the first and second stages take?

    Hint: Let x be the rate of oxygen pumped in by the oxygen pump, in modules-worth per hour. Likewise let y be the rate pumped out by the exhaust pump, in modules-worth per hour.

Answers

1. Let x be Team B. Then Team A is $x-1$ and likewise Team C is $x-3$. Thus we have $(x-1)+x+(x-3)=116$ or $3x-4=116$ and clearly $x=40$. Thus Team A sold 39 cars, Team B sold 40 cars, and Team C sold 37 cars.

2. Let x be the cheaper factory and then $3x$ is the more expensive one. We have $900=x+3x$ or $900=4x$, and that means $x=900/4=225$. Thus the cheaper factory is worth 225 million and the more expensive factory is worth 675 million. We can check that 225+675=900.

3. Suppose Alice has x and Bob has y. We see that $x+3=3y$ by the fact that triple Bob's balance isn't Alice's balance, but is off by three dollars.

In other words, if Alice had 3 dollars more, then she'd be triple Bob. We also see that $x + y = 1568.48$, because both balances come to 1568.48.

Now, let's solve the second equation for y to get $y = 1568.48 - x$ and plug this into the first equation. We obtain

$$\begin{aligned} x + 3 &= 3y \\ x + 3 &= 3(1568.48 - x) \\ x + 3 &= 4705.44 - 3x \\ 4x + 3 &= 4705.44 \\ 4x &= 4702.44 \\ x &= 4702.44/4 \\ x &= 1175.61 \end{aligned}$$

Then we can use $y = 1568.48 - x$ to compute $y = 1568.48 - 1175.61 = 392.87$. We can check our work with

$$1175.61 + 392.87 = 1568.48$$

as well as by checking that

$$3(392.87) = 1178.61 = 1175.61 + 3$$

as desired.

Option b for #3: Let x be Bob's balance, then Alice's balance is $3x - 3$. Therefore

$$\begin{aligned} x + (3x - 3) &= 1568.48 \\ 4x - 3 &= 1568.48 \\ 4x &= 1571.48 \\ x &= 392.87 \end{aligned}$$

and so $x = 392.87$ is Bob's balance while Alice's balance is

$$3(Bob) - 3 = 3(392.87) - 3 = 1178.61 - 3 = 1175.61$$

.

We can check with

$$1175.61 + 392.87 = 1,565.48$$

Option c for #3: Let x be what Alice has. Then $x + 3$ would be what Alice would have if she had 3 dollars more. If $x + 3$ is exactly triple what Bob has, then Bob has $(x + 3)/3$ right now, or $(1/3)x + 1$. Then together they have

$$\begin{aligned} x + (1/3)x + 1 &= 1568.48 \\ (4/3)x + 1 &= 1568.48 \\ (4/3)x &= 1567.48 \\ x &= (3/4)(1567.48) \\ x &= 1175.61 \end{aligned}$$

Now $x = 1175.61$ is what Alice has. Bob has $(1175.61)/3 + 1 = 392.87$. Last we can double check

$$1175.61 + 392.87 = 1565.48$$

which is exact to the penny.

4. Denote their goal as g. Right now, they have $g/6$. However, this is the same thing as $g/4 - 5900$, as they are 5900 dollars short of a quarter of their goal. Therefore, we write

$$\frac{g}{6} = \frac{g}{4} - 5900$$

and now I'll show you two ways of solving this equation. First, the standard way:

$$\begin{aligned} g/6 &= g/4 - 5900 \\ g/6 - g/4 &= -5900 \\ g\left(\frac{1}{6} - \frac{1}{4}\right) &= -5900 \\ g(-1/12) &= -5900 \\ g &= (5900)(-12) \\ g &= 70,800 \end{aligned}$$

However, if you dislike fractions, you can start by multiplying by 24. This turns out to be faster.

$$\begin{aligned} g/6 &= g/4 - 5900 \\ 4g &= 6g - 141,600 \\ 4g - 6g &= -141,600 \\ -2g &= -141,600 \\ g &= 70,800 \end{aligned}$$

Of course, both methods yield the same answer. Now we know that the charity's goal is \$ 70,800, and the charity has $70,800 \div 6 = 11,800$ at this moment. To check our work we see that $70,800 \div 4 = 17,700$ and we breathe a sigh of relief because $11,800 + 5900 = 17,700$.

5. Let x be the number of accidents on the milling machine. Then all the other machines combined made $x-8$. This means there were $(x)+(x-8) = 42$ accidents, which we can write $2x - 8 = 42$ accidents. This means that $2x = 42 + 8 = 50$ so clearly $x = 25$. We should write that the milling machine caused 25 accidents.

   To check our work, $25 - 8 = 17$ would be the number of accidents of all other types. This means that $25 + 17 = 42$ accidents occurred in total, and that matches the data in the problem.

6. Let x be the total in the pot. Then Alvin lost $0.2x$, while Simon lost $0.2x+100$, and Theodore lost $0.2x+160$. The total comes to $0.6x+260 = x$. Then $260 = 0.4x$, and thus $x = 650$. So Alvin lost \$ 130, while Simon lost \$ 230, and Theodore lost \$ 290. The winner (who has no name, in this problem) walks away with $130+230+290 = 650$.

7. Let x be what she charges in the summer months, and y be what she charges in the off-season months. Since 4 summer months are named, there are obviously 8 off-season months. Then a normal year would produce

$$4x + 8y = 15,060$$

but in the year 2010, due to fire damage, she only collects 3 summer months and 4 off-season months. Therefore we have

$$3x + 4y = 9105$$

Doubling the latter equation we have

$$6x + 8y = 18,210$$

and then subtracting the first equation from that we get

$$2x + 0y = 18,210 - 15,060$$

or then $2x = 3150$ which means $x = 1575$. Then we can plug that in to the first equation to get

$$4(1575) + 8y = 15,060$$

which means that $8y = 8760$ and therefore $y = 1095$. Now we should check our work, using the second equation, and get

$$3(1575) + 4(1095) = 4725 + 4380 = 9105$$

as desired.

8. Let x be the number of bottles at the start of the problem. Surely $8x$ is the value of the first offer, and so the second offer would be 23 more or $8x + 23$. That second offer consists of selling for \$ 9.50 per bottle, but there are only $x - 2$ bottles remaining. So it could also be written as $(9.50)(x - 2)$. Thus we have

$$\begin{aligned} 8x + 23 &= 9.50(x - 2) \\ 8x + 23 &= 9.50x - 19 \\ 8x + 42 &= 9.50x \\ 42 &= 1.50x \\ 28 &= x \end{aligned}$$

Thus the original batch had 28 bottles, and would have sold for $(28)(8) = 224$. The 26 now-remaining bottles sold for $(26)(9.50) = 247$. Sure enough, $247 - 224 = 23$, as required.

There is one last land-mine. Many students would triumphantly mark 28, but we're asked how many bottles were in the *final* sale, which is 26. One must always be very careful to answer exactly the question asked.

9. Suppose that x senior employees are in favor of the opponent. Then $x+400$ are in favor of the current boss. Then, with junior employees included, this becomes $x + 100$ in favor of the opponent, and $x + 500$ in favor of the current one. We know then that $2(x + 100) = x + 500$ or more simply, $2x + 200 = x + 500$, which means of course that $x = 300$. Thus without junior employees, the current boss will get 700 votes compared to 300.

Removed Problems: These last few problems were at one time removed because they are not taking place in a business-related setting. However, if you need more practice, then feel free to use them.

10. Let x be the cheaper coin and then $3x$ is the more expensive one. We have $880 = x + 3x$ or $x = 880/4 = 220$. Thus the cheaper coin is worth \$ 220 and the more expensive one is worth \$ 660. We can check with $220 + 660 = 880$, and indeed $3 \times 220 = 660$ which is also reassuring.

11. Suppose the commander started with x Canadians and $2x$ Brits. Then he ended with $x - 700$ Canadians and $2x - 700$ Brits. So we have

$$3(x - 700) = 2x - 700$$

and that becomes $3x - 2100 = 2x - 700$ or more simply $x = 1400$. He started with 1400 Canadians and 2800 Brits, then was reduced to 700 and 2100.

12. Suppose that there are x sophomores. We know that $x/2 - 29$ is the number with an off-campus parking permit and there are 1437 off-campus parking permits. Therefore, we have

$$\begin{aligned} x/2 - 29 &= 1437 \\ x/2 &= 1466 \\ x &= 2932 \end{aligned}$$

Therefore, we can infer that there are 2932 sophomores. To check our work, note that half of this is $2932/2 = 1466$ and if we subtract 29 we obtain $1466 - 29 = 1437$ as desired.

13. The key to this problem is Stage Three, when both pumps are running, and the room starts full and ends empty. Immediately from this we see that the exhaust pump is stronger/faster than the oxygen pump, because the exhaust pump "wins" (i.e. the end state is an empty module.)

If we wish to be very precise, we could write $x < y$ (the rate of the oxygen pump is lower/slower than the exhaust pump) and equivalently $1/x > 1/y$ (the filling of the module takes longer than the emptying) but as it comes to pass, this is unneeded.

During Stage Three, we have x coming in and y going out. So the net exhaust rate is $y - x$ per hour, or $12(y - x)$ in 12 hours. Since this is 1 modules-worth, we write

$$12(y - x) = 1$$

and now move to examine Stages Four and Five.

In Stage Four, we have only the oxygen pump on, and so oxygen is coming in at the rate of x modules-worth per hour. Therefore, $3x$ modules-worth comes in during Stage Four. During Stage Five, both pumps are running, so the rate of outgo is $y-x$ as in Stage Three. But this time there are nine hours, so $9(y - x)$ is removed. The module ends Stage Five empty, yet it starts out not full, but rather with $3x$ modules-worth in it. Therefore we have

$$9(y - x) = 3x$$

as our equation for Stage Five.

Now, after multiplying out, we have

$$\begin{aligned} 12y - 12x &= 1 \\ 9y - 9x &= 3x \end{aligned}$$

and should multiply the bottom equation by $4/3$ to obtain

$$12y - 12x = 4x$$

and subtract this from the first equation to get

$$(12 - 12)y + (-12 - -12)x = 1 - 4x$$

or more plainly

$$0y + 0x = 1 - 4x$$

so that we see $1 - 4x = 0$ or $1 = 4x$ therefore $x = 1/4$.

Plugging that back into the first equation, we have

$$12(y - 1/4) = 1$$

or that $y - 1/4 = 1/12$ therefore $y = 1/4 + 1/12 = 1/3$.

And since the exhaust pump removes $1/3$ modules-worth of air per hour, it empties the module in 3 hours, which is the length of Stage One. Likewise, since the oxygen pump inserts $1/4$ modules-worth of air per hour, it fills the module in 4 hours, which is the length of Stage Two.