Worksheet on
Word Problems about Mixed Commodities

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Questions

1. You are visiting one of the states without a sales tax, and two college students are in front of you at a convenience store. One buys 4 six-packs of the local microbrew beer, and 2 packages of potato chips, for \$ 39.30. The other buys 3 six-packs of the local microbrew beer, and 5 packages of potato chips, for \$ 39.10. How much is a six-pack of beer going to cost? How much is a bag of potato chips going to cost?

2. An apartment building has one-bedroom and two-bedroom apartments. The two-bedroom apartments rent for \$ 1500 and the one-bedroom apartments rent for \$ 1000. The total revenue for the building is \$ 49,500 per month. Next year, the rent on the one-bedrooms goes up by \$ 100, but on the two-bedrooms by \$ 200. If revenue goes up by \$ 5900, then how many bedrooms are there of each type? Note, neither year had any vacancies.

3. A small plant near Hull, Quebec is part of a company that is not doing well. They are ordered to lay off 200 employees. Before the layoff, the plant has twice as many francophone workers as anglophone workers.

   The HR office emails the plant manager, Jean-Pierre Petit, asking how to proceed in selecting employees for separation. In response, Jean-Pierre writes a long and flowery email stressing the importance of fairness, the history of cooperation between Canada's anglophone and francophone populations, the rich heritage of each community, and his own personal commitment to impartiality. In the spirit of fairness, he insists that the 200 laid-off employees comprise 100 anglophone and 100 francophone workers.

   A month later, the local newspaper reports that the plant now has three times as many francophones as anglophones.

   Let's see if we can map out what went wrong here...

(a) How many people work at the factory before the layoffs?

(b) How many people work at the factory after the layoffs?

(c) Before the layoffs, how many workers at the factory were anglophone? How about francophone?

(d) After the layoffs, how many workers at the factory are anglophone? How about francophone?

(e) What percentage of the workers before the layoffs were francophone?

(f) What percentage of the workers after the layoffs are francophone?

(g) Is there an arrangement to lay off 200 employees that would have kept the ratio between francophones and anglophones constant?

4. At the regional distribution center for a grocery store chain, the usual crew of the loading dock is on strike. Inexperienced workers have been brought in to do the heavy lifting, and junior-level managers such as yourself are supervising. One truck is going to be loaded with crates of oranges and strawberries. After loading the truck, which takes quite some time, you realize that you did not have anyone count how many crates of oranges and how many crates of strawberries were loaded, and this information is necessary for inventory tracking. To unload the truck, count, and reload the truck would be an expensive waste of time (and therefore wages), so naturally your boss orders you to compute the needed information with algebra.

   The truck is filled to capacity (in terms of volume) and has a net cargo weight of 7782 lbs. (Note, the net cargo weight is found by weighing the truck without cargo, loading it with cargo, weighing it again, and subtracting.)

   The truck has 300 cubic feet of cargo space. The orange crates are 1.5 cu feet but weigh 42 lbs, whereas the strawberry crates are 0.50 cu feet but weigh 12 lbs. How many of each type of crate is on board?

5. The accountant who handles the book keeping for a local theatre has come across some incidental information. A theatre sells tickets for the main floor at $ 17 each, and the mezzanine at $ 11 each. They have booked an exciting off-broadway performance. For the evening showing, they sold out, and the total revenue was $ 8430. For the afternoon showing, however, they sold a third of the mezzanine and half the main floor. The total revenue for the afternoon was $ 3830. Out of curiosity, is it possible (using this data) to compute how many seats there are on each level? If so, how many seats are there?

6. A fire has destroyed the inventory of the local computer store and also has incinerated many of the records. The insurance adjustor has to do some mathematics and some critical thinking to figure out how much actually was destroyed by the fire.

For the sake of simplicity, let us assume that the store sold only two models of one computer. The "regular" version sold for \$ 1995 and the "pro" version sold for \$ 2495. The value of the store's inventory at the start of the quarter was \$ 890,400 dollars, however, 60% the regular computers and 40% of the pro computers had sold at the time of the fire. The revenue since the start of the quarter can be found on the store's bank's website, and totals \$ 481,845 dollars. How many of each computer got destroyed in the fire and what is the total dollar value of the losses?

7. A company has two products, propeller blades and jet-engine blades. The propeller blades require 15 minutes in Machine X, and 30 minutes in Machine Y. The jet-engine blades require 45 minutes in Machine X, and 20 minutes in Machine Y. How many of each must be manufactured each day, to ensure that both machines are fully utilized? The manager of the factory wants both machines to run for 17.5 hours each.

8. A housing developer has bought some large parcel of land, and built several houses on it. She is now selling the houses individually to customers. The houses come in two sizes, selling at \$ 300,000 and \$ 400,000 each. Your realtor does not know how many of each kind there are, but he strongly believes that two-thirds of the cheaper houses, and one-third of the more expensive houses, have already sold. Using some information from some websites, he can determine that the fair market value of the entire development is \$ 20.1 million, and that so far the total sales has been \$ 10.6 million. How many of each type of house is there in the development?

9. Sheila has stock in General Mechanics (49 shares) and also in Abysmal Airlines (81 shares). Yesterday, her portfolio was worth \$ 16,270. At the end of trading today, General Mechanics was up 1% while Abysmal was down 2%. However, she was only down \$ 68.15, so she doesn't really mind. At what prices did General Mechanics and Abysmal Airlines trade for, both today and yesterday?

10. On January 1st, an investor has bought \$ 10,000 worth of stock in two companies: IBN and GN. The price of IBN was \$ 41 per share on the 1st of January, and GN was \$ 29 per share. The dividends for the year came to \$ 479.45, (which is rather good by the way). The dividend was \$ 2.05 per share for IBN and \$ 1.30 per share for GN. How much did she buy of each company?

11. (Warning: this one is sort of hard.) A public university classifies students as in-state or out-of-state. This university has two summer sessions each summer. For summer classes in the first session, there were two times as many in-state as out-of-state students. All the students from the first summer session stayed for the second summer session. Also, 50 new out-of-state students students joined for the second session, as well as 350 new in-state students. Now there are three times as many in-state students

as out-of-state students in the second summer session. Find the total enrollment in both sessions, for both in-state and out-of-state students.

Note: The remaining problems in this worksheet have been solved using the single-variable method in the answer section. However, if you prefer the two-variable method, then you can go ahead and solve them that way. The actual numerical answers at the end should be the same. If you want to try the single-variable method, to help you organize your work, consider the following hints:

- In Problem # 12 we have computer games, some at \$ 19.95 and some at \$ 24.95.
- In Problem # 13 we also have tickets, this time divided into student tickets and "other" tickets.
- In Problem # 14 we have silver, which is divided into 99.9%-pure and 92.5%-pure grades.
- In Problem # 15 we have coal, which is divided into higher-quality coal and lower-quality coal.
- In Problem # 16 we have hours worked during a summer job, which are divided into hours worked at a lower salary before a promotion, and hours worked at a higher salary after a promotion.
- In Problem # 17 we have hours worked tutoring mathematics, which are divided into hours with an annoying client, and hours with all other clients.
- In Problem # 18 we have miles driven on a long trip, divided into miles "with" and "without" air conditioning.
- In Problem # 19 we have regular tickets and tickets sold to students/seniors.

The secret in each of these problems is to make one of those subcategories x, and the other subcategory usually becomes "some number minus x."

12. A group of computer games is for sale at the price of \$ 19.95, and others are for sale at the price of \$ 24.95. If Louis buys a dozen games for \$ 274.40, then how many computer games of each price did he buy?

13. Tickets to a local football game recently sold for \$ 15 per student, and \$ 25 for all others. The total revenue was \$ 292,920, and 17,200 tickets were sold. How many student tickets were sold and how many "other" tickets were sold?

14. A jewelry supply store has a stockpile of fine silver (99.9% pure) and a stockpile of sterling silver (92.5% pure). Fine silver is expensive compared to sterling silver. A client wants 95%-pure silver for making some large pieces. Now, of course, the supply store could sell the client the 99.9%-pure silver, but that's a bit of a waste, as it is a higher quality than what

the client wants. Suppose 15 kg of 95%-pure silver is ordered. How much of each type of silver should be mixed together?

15. A coal-processing plant wants to make some medium-quality coal, which has a fair-market value of 170 dollars per metric ton. They have 60 metric tons of higher-quality coal on hand, costing 175 dollars per metric ton, and they want to mix it all with however many tons they need of lower-quality coal. The lower-quality coal has a fair-market value of 160 dollars per metric ton. How many tons of lower-quality coal should they mix in to achieve this result? (Try it now, and if you get stuck, a hint is given below.)

Hint: If you get stuck on the previous problem, your first line could be, "Suppose they combine x tons of lower-quality coal to mix with the 60 tons of higher-quality coal to get $60+x$ tons of medium-quality coal." Then compute the fair-market value of the lower-quality portion, the higher-quality portion, and the medium-quality mix.

16. Two twins, Rebecca and Leah, are working at the local grocery store. They both start at \$ 8 per hour, but after some time Rebecca gets promoted and earns \$ 11 per hour from then on. They work 480 hours each over the entire summer. The total earned by the twins (before taxes and other withholdings) is \$ 8520. How many weeks does Rebecca work at each pay rate? (You may assume each sister worked a standard 40-hour work week.)

17. Two math tutors—husband and wife—normally both charge \$ 50 per hour. Together they earned \$ 3080 this past summer. Both worked exactly 30 hours, but one of them had a particularly irritating client who was charged \$ 60 per hour. How many hours were spent with the irritating client?

18. Note, this one is very challenging! After many months, a professional delivery driver knows that he can get 21 miles per gallon without running the air conditioning and 18 miles per gallon while running the air conditioning. It is September and he's been hired to make an emergency overnight delivery of spare parts from a factory in Chicago to a client in Mexico City. He gets paid a flat fee and must purchase his own gas, so to save gas money he starts out with no air-conditioning. The entire trip was 2043 miles, but he was not able to forego using the AC and eventually switched it on when he could stand the heat no longer. Now that the trip is over, he knows that he used 104 gallons of gas, but he'd like to know how many miles he drove without AC and how many miles he drove with AC.

19. Suppose tickets to a play are going for \$ 55 each, except for seniors and students, which sell for \$ 35. If there were 161 tickets sold for a total of \$ 8095, how many of each type were sold?

Answers

1. Let c be the number of bags of potato chips, and let b be the number of six-packs of beer. From the first purchase we learn that

   $$4b + 2c = 39.30$$

   while from the second purchase we learn that

   $$3b + 5c = 39.10$$

   Let's multiply the first equation by 2.5 and obtain

   $$10b + 5c = 98.25$$

   and then subtract the second equation from that one to obtain

   $$7b + 0c = 59.15$$

   and because we know that $7b = 59.15$, we can just divide by 7 and get $b = 8.45$.

   If we plug this back into the first equation we get

   $$4(8.45) + 2c = 39.30$$

   which, after distributing, turns into

   $$33.80 + 2c = 39.30$$

   and then we subtract the 33.80 to get $2c = 5.5$. Finally, we divide by two and get $c = 2.75$.

   We now know both the value for b and the value for c. Are we done? Nope! Now, let's check our work by plugging b and c back into the second equation. Since we used the first equation to find c, it is better to use the second equation to check. We get

   $$3(8.45) + 5(2.75) = 25.35 + 13.75 = 39.10$$

   which is exact, to the penny.

2. Let x be the number of one-bedroom apartments and y be the number of two-bedroom apartments. Since the rents are 1000 and 1500, and revenue is 49,500, we have

   $$1000x + 1500y = 49,500$$

   as the first equation. In the second year, we have 1100 and 1700 for the rents, and a revenue of 55,400. This yields the equation

   $$1100x + 1700y = 55,400$$

and multiplying the first equation by 1.1 provides

$$1100x + 1650y = 54,450$$

and now we can subtract that from the second equation to obtain

$$0x + 50y = 950$$

which means that $y = 950/50 = 19$. Then we plug this back into the first equation to get

$$1000x + 1500(19) = 49,500$$

and subtracting $1500(19) = 28,500$ from each side we obtain

$$1000x = 21,000$$

and so then clearly $x = 21$. We can check this with the second equation

$$1100(21) + 1650(19) = 23,100 + 31,350 = 54,450$$

exact to the penny!

3. Most problems in this problem set can be solved using one-variable linear equations—and that includes this one. However, it is easier to use two variables and to intersect two lines. I show both methods—the easier first, and then afterward, the single-variable version.

   Suppose there are a anglophones working at the factory at the start of the problem. Then there are f francophones. After the layoffs, there will be $a-100$ anglophones, and $f-100$ francophones. Using the before-and-after ratios, we have

$$\begin{aligned} 2a &= f \\ 3(a-100) &= f-100 \end{aligned}$$

   We can plug $f = 2a$ into the second equation to get

$$\begin{aligned} 3(a-100) &= 2a-100 \\ 3a-300 &= 2a-100 \\ 3a &= 2a+200 \\ a &= 200 \end{aligned}$$

   Now we know $a = 200$ and $f = 400$. This allows us to say:

   (a) Before the layoffs, there are 600 workers at the factory.
   (b) After the layoffs, there are 400 workers at the factory.
   (c) Before the layoffs, there were 200 anglophones and 400 francophones.

(d) After the layoffs, there are 100 anglophones and 300 francophones.

(e) Before the layoffs, $66.6\overline{6}\%$ of the employees were francophone.

(f) After the layoffs, 75% of the employees were francophone.

(g) No. Using a direct proportion, you get $66.6\overline{6}$ and $133.3\overline{3}$. The closest you could get would be 66 anglophone and 134 francophone, or alternatively, 67 anglophone and 133 francophone. It is not possible to keep the ratio constant.

Alt. algebra: The factory started with a anglophones and $2a$ francophones before the layoffs. After the layoffs, they have $a - 100$ anglophones and $2a - 100$ francophones. Then we write

$$\begin{aligned} 3(a-100) &= 2a - 100 \\ 3a - 300 &= 2a - 100 \\ 3a &= 2a + 200 \\ a &= 200 \end{aligned}$$

Now we know they started with 200 anglophones and 400 francophones. This allows us to say:

(a) Before the layoffs, there are 600 workers at the factory.

(b) After the layoffs, there are 400 workers at the factory.

(c) Before the layoffs, there were 200 anglophones and 400 francophones.

(d) After the layoffs, there are 100 anglophones and 300 francophones.

(e) Before the layoffs, $66.6\overline{6}\%$ of the employees were francophone.

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4. Let x be the number of orange crates and y be the number of strawberry crates. We know that the truck is filled to capacity in terms of volume, and the volume is 300 cubic feet. Since each orange crate is 1.5 cubic feet and each strawberry crate is 0.5 cubic feet, we would write down

$$1.5x + 0.5y = 300$$

and then examine the weight. We learn that orange crates weigh 42 lbs while strawberry crates weigh 12 lbs. The total weight is 7782 pounds, so we write down

$$42x + 12y = 7782$$

Now, let's multiply the first equation by 24. We obtain

$$36x + 12y = 7200$$

and we can subtract this from the second equation to get

$$6x + 0y = 582$$

which means that $x = 582/6 = 97$. Next we plug that into the first equation to obtain

$$(1.5)(97) + 0.5y = 300$$

which becomes

$$145.5 + 0.5y = 300$$

which means that

$$0.5y = 154.5$$

and finally we multiply by two to learn that $y = 309$.

Last but not least we check our work with the second equation

$$(42)(97) + (12)(309) = 4074 + 3708 = 7782$$

which is absolutely correct. Finally, we write down that there are 309 crates of strawberries and 97 crates of oranges.

5. Let the number of seats in the mezzanine be m and the number of seats on the floor be f. Then, for the night that was sold out, we have

$$17f + 11m = 8430$$

but for the matinee we have

$$17(f/2) + 11(m/3) = 3830$$

which is easier written as

$$\frac{17}{2}f + \frac{11}{3}m = 3830$$

Now we can divide the first equation by two, resulting in

$$\frac{17}{2}f + \frac{11}{2}m = 4215$$

and we can subtract the third equation from this to get

$$0f + \left(\frac{11}{2} - \frac{11}{3}\right)m = 4215 - 3830$$

which turns into

$$\left(\frac{11}{6}\right)m = 385$$

and finally we conclude with

$$m = 385\left(\frac{6}{11}\right) = 210$$

To find f, we plug this into the first equation and get

$$17f + (11)(210) = 8430$$

which becomes

$$17f = 8430 - 2310 = 6120$$

and we conclude that $f = 6120/17 = 360$.

Last but not least we check our work with the second equation

$$17\frac{360}{2} + 11\frac{210}{3} = 3060 + 770 = 3830$$

which is exact, to the penny.

6. Let r be the number of regular computers at the start of the quarter and let p be the number of pro computers at the start of the quarter. Because the inventory is \$ 890,400 we know that

$$1995r + 2495p = 890,400$$

and then from the revenue data, we know that $0.6r$ regular computers got sold and $0.4p$ pro computers got sold. This means that

$$1995(0.6r) + 2495(0.4p) = 481,845$$

which simplifies to

$$1197r + 998p = 481,845$$

Multiplying the first equation by 0.4 yields

$$798r + 998p = 356,160$$

and then we can subtract this from the second equation to obtain

$$399r + 0p = 125,685$$

and we divide both sides by 399 to get $r = 315$. Plugging this back into the first equation will result in

$$1995(315) + 2495p = 890,400$$

which turns into

$$628,425 + 2495p = 890,400$$

or more simply

$$2495p = 261,975$$

which we divide by 2495 to obtain $p = 105$.

Before we continue, let's check this with the second equation:

$$1197(315) + 998(105) = 377,055 + 104,790 = 481,845$$

which is exact!

Now we know that the quarter started with 315 regular computers. However, we are asked to find out how many were destroyed in the fire. We know that 60% of them sold, which is $(0.6)(315) = 189$, and so 40% did not sell, which is $(0.4)(315) = 126$. Just to check $189+126 = 315$, which is good news. Similarly, we know that 40% of the pro model computers sold, which is $(0.4)(105) = 42$ and that means 60% of the pro model computers were destroyed in the fire. That comes to $(0.6)(105) = 63$. Again, just to be paranoid, $42 + 63 = 105$, which is good news.

Therefore, we can tell the insurance company that 63 pro computers and 126 regular computers were destroyed in the fire. This means that the insurance company should, for the inventory alone, pay the store owner

$$(63)(2495) + (126)(1995) = 157,185 + 251,370 = 408,555$$

Finally, we can check all of this work with

$$481,845 + 408,555 = 890,400$$

which means that every dollar of the total value of the inventory at the start of the quarter can be found either in the revenue obtained by selling some computers, or in the dollar amount destroyed by fire. It is always nice when everything balances out.

7. The trick here is to convert "17.5 hours" into minutes. We have

$$17.5 \times 60 = 1050$$

Now suppose we have p propeller blades and j jet-engine blades. Then for Machine X we require

$$15p + 45j = 1050$$

and for Machine Y we require

$$30p + 20j = 1050$$

Now we can double the first equation to get

$$30p + 90j = 2100$$

and now subtract from this the second equation to obtain

$$0p + 70j = 1050$$

and so we have $j = 1050/70 = 15$.

Then we plug this into the first equation to get

$$15p + 45(15) = 1050$$

which becomes

$$15p + 675 = 1050$$

and so we have $15p = 375$ or $p = 375/15 = 25$.

Now we can check our work using the second equation

$$(30)(25) + (20)(15) = 750 + 300 = 1050$$

which matches perfectly.

We conclude that the manufacturer should produce 15 jet-engine blades and 25 propeller blades per day.

8. Let's do this problem in thousands of dollars, rather than in dollars. Let c be the number of cheaper houses and x be the number of expensive houses. The fair market value of the entire development is 20.1 million dollars, so we can write

$$300c + 400x = 20,100$$

We know that $(2/3)c$ of the cheaper houses have sold and $(1/3)x$ of the expensive houses have sold, generating 10.6 million in sales. Therefore, we write

$$300(2/3)c + 400(1/3)x = 10,600$$

which becomes

$$200c + (400/3)x = 10,600$$

and if we multiply that by (3/2) we would obtain

$$300c + 200x = 15,900$$

We'll subtract the fourth equation from the first equation to obtain

$$0c + 200x = 4200$$

and that implies $x = 4200/200 = 21$.

Plugging that into the first equation yields

$$300c + (400)(21) = 20,100$$

and that becomes

$$300c = 20,100 - 8400 = 11,700$$

which means that $c = 11,700/300 = 39$.

We can check our work with the second equation

$$300(2/3)(39) + 400(1/3)(21) = 7800 + 2800 = 10,600$$

and that's an exact match!

9. Suppose that General Mechanics was trading for g dollars per share yesterday, and therefore $1.01g$ dollars per share today. Likewise suppose that Abysmal Airlines was trading for a dollars per share yesterday and thus $0.98a$ dollars per share today. Then we have

$$49g + 81a = 16,270$$

for yesterday, and

$$49(1.01)g + 81(0.98)a = 16,270 - 68.15$$

for today. We can massage the second equation to get

$$49.49g + 79.38a = 16,201.85$$

but then if we multiply the first equation by 1.01 (in order to turn that 49 into 49.49) we would have

$$49.49g + 81.81a = 16,432.70$$

and then subtracting the two equations yields

$$(49.49 - 49.49)g + (81.81 - 79.38)a = 16,432.70 - 16,201.85$$

which becomes

$$2.43a = 230.85$$

or equivalently $a = 230.85/2.43 = 95$. Now we substitute back into the original

$$49(g) + 81(95) = 16,270$$

which becomes

$$49g + 7695 = 16,270$$

and thus

$$49g = 16,270 - 7695 = 8575$$

and so $g = 175$. Thus General Mechanics traded for 175 dollars per share yesterday, rising to $175(1.01) = 176.75$ dollars per share today. Meanwhile, Abysmal Airlines fell from 95 dollars per share to 93.10 dollars per share.

Surely after such a long problem it is necessary that we check our work. And so we check that

$$49(175) + 81(95) = 8575 + 7695 = 16,270$$

and also that

$$49(176.75) + 81(93.10) = 8,660.75 + 7,541.10 = 16,201.85$$

and finally

$$16,270 - 16,201.85 = 68.15$$

as desired!

10. Let i indicate the number of shares of IBN and g be the number of shares of GN. The equations are obviously

$$\begin{aligned} 41i + 29g &= 10,000 \\ 2.05i + 1.30g &= 479.45 \end{aligned}$$

multiplying the second one by 20 yields

$$\begin{aligned} 41i + 29g &= 10,000 \\ 41i + 26g &= 9589 \end{aligned}$$

and subtracting gives

$$0i + 3g = 411$$

which means $g = 137$. Plugging this back into the original first equation gives

$$\begin{aligned} 41i + 29(137) &= 10,000 \\ 41i + 3973 &= 10,000 \\ 41i &= 6027 \\ i &= 147 \end{aligned}$$

and so we learn that she had 137 shares of GN and 147 shares of IBN. We can check this with

$$2.05(147) + 1.30(137) = 301.35 + 178.1 = 479.45$$

which is exact, to the penny.

11. (Warning: this one is hard.) In the first session, we start with x out-of-state students and $2x$ in-state students. Then in the second session, we have $x + 50$ out-of-state students, and $2x + 350$ in-state students.

Since there are three times as many in-state students as out-of-state students, we write

$$\begin{aligned} 3(\text{out-of-state}) &= \text{in-state} \\ 3(x+50) &= 2x+350 \\ 3x+150 &= 2x+350 \\ x+150 &= 350 \\ x &= 200 \end{aligned}$$

Therefore, in the first summer session, there are 200 out-of-state students and 400 in-state students. In the second summer session, there are 250 out-of-state students and 750 in-state students.

12. Let x be the number of games bought at the price of \$ 19.95, and $12-x$ be the number of games bought at the price of \$ 24.95. Then we have

$$\begin{aligned} 19.95(x)+24.95(12-x) &= 274.40 \\ 19.95x+299.4-24.95x &= 274.40 \\ -5x+299.40 &= 274.40 \\ -5x &= 274.40-299.40 \\ -5x &= -25 \\ x &= 5 \end{aligned}$$

Therefore, we compute that Lewis bought 5 games at the lower price and $12-5=7$ at the higher price. Let's check this really quickly with

$$(5)(19.95)+(7)(24.95) = 99.75+174.65 = 274.40$$

which is exact to the penny.

13. Let x signify the number of student tickets, and $17,200-x$ signify the number of "other tickets." Then we have

$$\begin{aligned} 15x+(25)(17,200-x) &= 292,920 \\ 15x+430,000-25x &= 292,920 \\ 430,000-10x &= 292,920 \\ -10x &= -430,000+292,920 \\ -10x &= -137,080 \\ x &= 13,708 \end{aligned}$$

At this point, we have computed $x = 13,708$ student tickets which would require $17,200-13,708 = 3492$ other tickets. Now we can check our work:

$$(15)(13,708)+(25)(3492) = 205,620+87,300 = 292,920$$

which is absolutely exact.

14. Suppose there are x grams of fine silver and $15,000 - x$ grams of sterling silver. Then we'd have

$$\begin{aligned} (x)(0.999) + (15,000 - x)(0.925) &= (15,000)(0.95) \\ 0.999x + 13,875 - 0.925x &= 14,250 \\ 0.074x &= 375 \\ x &= 375/0.074 \\ x &= 5067.\overline{567} \end{aligned}$$

This would imply using an amount of sterling equal to

$$(15,000 - 5067.\overline{567}) = 9932.\overline{432} \text{ grams}$$

and we can check our work with

$$(5067.\overline{567})(0.999) + (9932.\overline{432})(0.925) = 5062.5 + 9187.5 = 14,250$$

comparing that to

$$(15,000)(0.95) = 14,250$$

an exact match.

15. We start with the hint: "Suppose they use x tons of lower-quality coal to mix with the 60 tons of higher-quality coal to get $60 + x$ tons of medium-quality coal." Now let's calculate the fair-market value of each.

The higher-quality coal is worth $(60)(175) = 10,500$ while the lower-quality coal is worth $(x)(160) = 160x$. Similarly, the medium-quality coal should be worth $(60 + x)(170) = 10,200 + 170x$. We get

$$\begin{aligned} (60)(175) + (x)(160) &= (60 + x)(170) \\ 10,500 + 160x &= 10,200 + 170x \\ 10,500 &= 10,200 + 10x \\ 300 &= 10x \\ 300/10 &= x \\ 30 &= x \end{aligned}$$

We can check our work:

$$(60)(175) + (30)(160) = 10,500 + 4800 = 15,300$$

is the cash value of the new coal mix, and that comes to

$$15,300/90 = 170$$

exactly as desired! Therefore, the manager of the coal plant should order that 30 tons of lower-quality coal be mixed with the 60-tons of higher quality coal.

16. The trick is to remove Leah from the problem. She worked 480 hours in the summer, and at \$ 8 per hour, that comes to $480 \times 8 = 3840$. This means that $8520 - 3840 = 4680$ was earned by Rebecca.

    Now that Leah has been removed from the problem, suppose Rebecca worked x hours at the lower wage, and $480 - x$ hours at the higher wage. Then she'd have earned $(8)(x)+(11)(480-x)$ dollars, before withholdings and taxes. We have

$$\begin{aligned} 8x + 11(480 - x) &= 4680 \\ 8x + 5280 - 11x &= 4680 \\ 5280 - 3x &= 4680 \\ -3x &= -600 \\ x &= 200 \end{aligned}$$

    This implies that she spent 200 hours at the lower pay rate, and therefore $480 - 200 = 280$ hours at the higher pay rate. We can check our work with

$$(200)(8) + (280)(11) = 1600 + 3080 = 4680$$

    which is exact!

    Now, there is another trap! We were asked to respond in weeks, not hours. So instead of 200 hours, we should say 5 weeks at the lower rate. Likewise, instead of 280 hours, we should say 7 weeks at the higher rate.

17. Suppose there were spent x hours with the irritating client, and then $60-x$ with the others clients. Then we have

$$\begin{aligned} 60x + 50(60 - x) &= 3080 \\ 60x + 3000 - 50x &= 3080 \\ 10x + 3000 &= 3080 \\ 10x &= 80 \\ x &= 8 \end{aligned}$$

    Thus, there were 8 hours with the irritating client, and 52 hours with the others.

    Let us check our work, by checking how much was paid. We obtain

$$(8)(60) + (52)(50) = 480 + 2600 = 3080$$

    which is exact.

18. Suppose x miles were driven without air-conditioning, and $2043 - x$ miles were driven with air-conditioning. Then the gallons of fuel used would have been $x/21$ for the miles without air-conditioning, and $(2043 - x)/18$

for the miles with air-conditioning. The total gallons of fuel used came to 104 gallons, so we write

$$\underbrace{x/21}_{\text{with AC}} + \underbrace{(2043 - x)/18}_{\text{w/o AC}} = \underbrace{104}_{\text{total}}$$

and proceed to solve

$$\begin{aligned}
x/21 + (2043 - x)/18 &= 104 \\
x + (21/18)(2043 - x) &= (104)(21) \\
18x + (21)(2043 - x) &= (104)(21)(18) \\
18x + 42,903 - 21x &= 39,312 \\
42,903 - 3x &= 39,312 \\
3591 - 3x &= 0 \\
3591 &= 3x \\
1197 &= x
\end{aligned}$$

Thus we would be tempted to conclude that $x = 1197$ miles were driven without air-conditioning and $2043 - x = 2043 - 1197 = 846$ miles were driven with air-conditioning. However, we should check our work. At 21 mpg, the 1197 miles without air-conditioning would require $1197 \div 21 = 57$ gallons. Likewise, at 18 mpg, the 846 miles with air-conditioning would require $846 \div 18 = 47$ gallons. Now we can see that $47 + 57 = 104$ gallons, as desired.

19. Let x be the number of discount tickets, then $161 - x$ are the full price tickets. This means that $35x + 55(161 - x) = 8095$ and that becomes $-20x + 8855 = 8095$ and therefore $x = 38$. So there are 38 discount tickets and $161 - 38 = 123$ regular tickets.