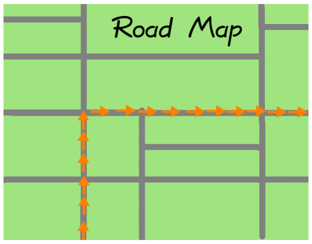
Module 3.1: Predatory Lending and the Annual Effective Rate



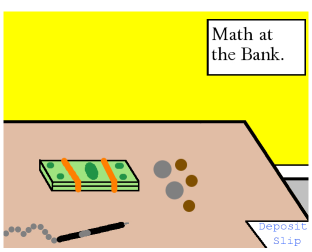
In this module, we are interested in computing an important number that reflects how severe or mild a loan's interest happens to be. This number has several names: the annual effective rate (AER), the annual equivalent rate (also AER), the effective annual rate (EAR), or just the effective rate (EFF). Another name is CAGR (Compounded Annual Growth Rate) used to properly evaluate investments instead of loans.

Whether you call it AER, EAR, EFF, or CAGR, this technique is particularly useful for revealing loans that appear to be moderate, but which are actually very severe, because the loan is using fees or commissions. Profiting from hidden fees is a common tactic in predatory lending.

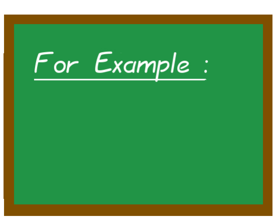
We'll also see an improper and useless way to evaluate investments, called the ROI (Return on Investment), and explore why it is improper and useless.

Before I begin, I'd like to discuss something with you. During this module, I'm going to show you the technique for computing this very important number, called the AER and many other names. The technique is not long and it is not all that difficult mathematically. However, it is a "procedure" and not a "formula."

There are some shortcut formulas for computing the AER. However, these formulas are very damaging and should not be used. The reason for that is that the shortcut formulas ignore fees. In real life, loans have fees. Therefore, we're going to learn how to compute the AER properly, gain facility with the procedure, and save the shortcut formulas until Page 421.



Some of the most common loans a person may take out in their lifetime are college loans, car loans, a mortgage for the house, small business loans, and signature loans. The first few might be familiar to you, but perhaps not the last one. *Signature loans* are made with no collateral at all. Collateral is property that the bank can seize if you fail to pay back the loan. In the case of a mortgage, the bank takes your house away; for a small business loan, there is real estate or some large piece of equipment that can be seized; with a car loan, the car can be seized. With a signature loan, you are pledging your good name and credit—if you fail to pay, the bank will destroy your credibility by contacting the three major credit bureaus.



3-1-1

Pat is shopping for a loan. He needs \$ 10,000 for some personal expenses, and so he is looking for a signature loan. One bank offers $16\frac{1}{8}\%$ compounded monthly, and the other offers $16\frac{1}{4}\%$ compounded quarterly. Which is the better loan? That is, which is the loan with the lower total payment? Pat hopes to pay off the loan in 1 year.

In both cases, $P = 10{,}000$ and $t = 1$. For the first loan, $r = 0.16125$ and $i = r/12 = 0.0134375$. (Remember, interest rates are hyper-sensitive to rounding error, and so we keep all the digits that our calculator will produce for interest rates. See the box with a bomb on Page 281). Then, because the loan is monthly $m = 12$, and $n = 12$ also, because we are paying the loan off in one year. Therefore we have

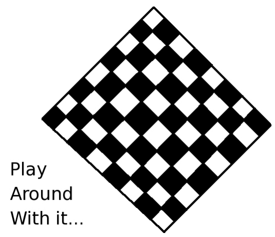
$$A = (10{,}000)(1.0134375)^{12} = 11{,}737.17$$

for the first loan, and we'll take care of the second loan in the next box.

Continuing with the previous box, we're going to compute Pat's other loan. For the second loan, $r = 0.1625$ and $i = r/4 = 0.040625$. Then, because the loan is quarterly $m = 4$ and $n = 4$. Therefore, we have

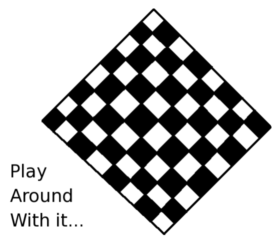
$$A = (10,000)(1.040625)^4 = 11,726.73$$

The difference isn't very large, in fact it is only \$ 10.44, but it is interesting that the quarterly loan, with a higher rate, is actually going to result in Pat paying less interest (\$ 1726.73 as compared with \$ 1737.17). Therefore, we learn that you cannot just compare rates when comparing loans.



3-1-2

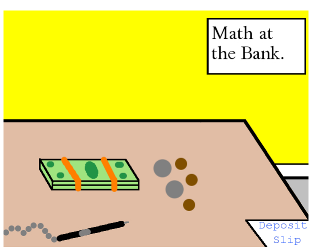
Suppose that Pat's friend Xavier also needs signature loans, but over a longer period of time, such as five years. We want to compute how things would change if the loans from the previous box were paid off in 5 years, instead of 1 year. Use the same interest rates as the previous box. What is the amount due in each case? Moreover, what is the difference between the monthly and quarterly loans?
[Answer: Monthly compounded loan = \$ 22,275.02, Quarterly compounded loan = \$ 22,176.09, and the difference is 98.93, a bit more interesting.]



3-1-3

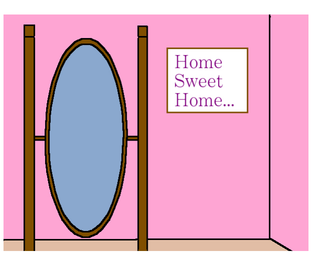
Now consider also $16\frac{3}{4}\%$ compounded annually, with the other two 5-year loans from the previous box. What is the amount due? Which of the three loans is best for Xavier? [Answer: Annually compounded loan = \$ 21,691.24.]

Despite the higher published rate, the last option (annually compounded), is the best. The published rate, which we've called r throughout our study of compound interest, is sometimes called the *nominal rate*.



Re-examine the previous box. Isn't that odd? The loan with the highest rate ($16\frac{3}{4}\%$, versus $16\frac{1}{4}\%$ and $16\frac{1}{8}\%$) ends up being the cheapest. We know this is true because Xavier will pay \$ 21,691.24 for the $16\frac{3}{4}\%$-annual loan, versus 22,176.09 for the $16\frac{1}{4}\%$-quarterly loan and \$ 22,275.02 for the $16\frac{1}{8}\%$-monthly loan.

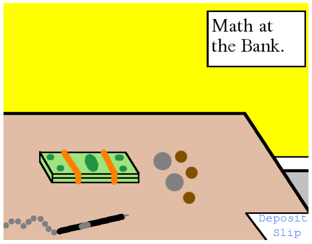
This surprising outcome is possible because there is more to a loan than just its interest rate! That is precisely the point of this module: we're going to learn how to calculate which loans are best.



A Pause for Reflection...

How does it come to pass, in rare circumstances, that a higher interest-rate loan could have you paying less interest than the lower interest-rate loan? Do you find this counter-intuitive?

Of course, what it comes down to is that a loan has many details by which it must be described. This includes the interest rate, of course, but there is also the duration and how frequently the loan is compounded. Soon, we will complicate matters further, by taking into account the fees of a loan.



Before we continue, it might be worth noting that the interest rates for Pat's and Xavier's signature loans might appear to be higher than what you're used to. As we said on the previous page, many loans have collateral, such as home mortgages or car loans. Because other loans do not, this is a disadvantage for banks.

As we said before, if you fail to pay your mortgage, the bank can seize your house and sell it; if you fail to pay your car loan, your car will be repossessed. For loans with no collateral, which include signature loans and credit cards, there is nothing to take. For this reason, they charge a higher interest rate, so that they get more money from those who will actually repay, to offset the losses from those who don't pay it back. The only thing backing a signature loan is your signature—it is literally a loan on your good name. Therefore, these loans often have rates that are triple that of a mortgage or car loan.

I'm going to claim now that Xavier's loan from a few boxes ago, that compounds monthly at $16\frac{1}{8}\%$ for 5 years, is equivalent to a loan which compounds annually at the rate of 17.3717715%. That's perhaps excessively precise, but as mentioned earlier interest rates are hypersensitive to rounding error. That hypersensitivity comes from the fact that exponents are involved, including large exponents on occasion.

We are stepping up from 6 digits of precision (used throughout this book) to 9 digits, just for this one problem. We'll return to 6 digits for the bulk of the module.

Let's verify my claim, by comparing the five year amount. We have

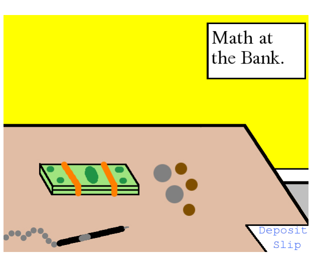
$$A = (10,000)(1 + 0.173717715)^5 = 22,275.02\cdots$$

which is exact to the penny.

Now I'm going to claim that Xavier's quarterly loan of $16\frac{1}{4}\%$ for 5 years is equivalent to an annual loan at the rate of 17.2673292%. Let's verify my claim again. Thus it is

$$A = (10,000)(1 + 0.172673292)^5 = 22,176.09\cdots$$

which is also exact to the penny.

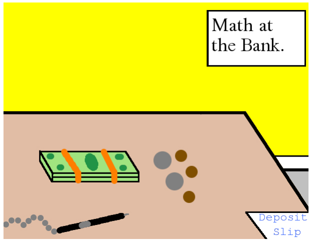


We can actually draw a financial conclusion from the last two boxes. One loan, compounded monthly, is equivalent to 17.3717715% while the other loan, compounded quarterly, is equivalent to 17.2673292%. Surely, all of us can easily see that

$$17.3717715 > 17.2673292$$

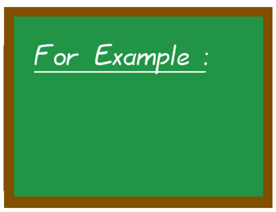
and therefore the quarterly loan is better for Xavier than the monthly loan. Now we must recall that the annual-compounding loan was $16\frac{3}{4}\%$.

Because these two very precise percentages are annual, and are both bigger than $16\frac{3}{4}\%$, we can see that the $16\frac{3}{4}\%$ loan is the best. After all, if they are all annual loans, and all for five years, then surely the lowest rate is the best loan, because there are no other features to compare—as you can see, all other factors have been held constant.



You might be wondering how I was able to find the numbers 17.3717715% and 17.2673292% when analyzing Xavier's loans several boxes ago. These interest rates are the *AER* or Annual Equivalent Rate for the loans which were discussed.

We will spend the bulk of this module learning how to compute the AER. Synonyms for the *Annual Equivalent Rate* include the *Annual Effective Rate*, the *Effective Annual Rate* (*EAR*), the *Effective Rate of Interest* (*EFF*), the Annual Percentage Yield (*APY*), and the *Compound Annually Growth Rate* (*CAGR*).



3-1-4

What is the annual equivalent rate (AER) of Xavier's loan that compounds monthly at $16\frac{1}{8}\%$, for a duration of five years?

To solve this, we're going to imagine two loans. The real loan is $16\frac{1}{8}\%$ compounded monthly, while the hypothetical loan has unknown rate and is compounded annually. The real and hypothetical loans go from the same P to the same A, in the same amount of time. Therefore, for both loans, we have $A = 22,275.02$ and $P = 10,000$, as well as $t = 5$.

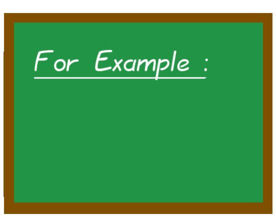
The hypothetical loan is annual, so $m = 1$. It is easy to compute

$$n = mt = (1)(5) = 5$$

and since $i = r/m$ we have $i = r/1 = r$. We can now compute

$$\begin{aligned}
A &= P(1+i)^n \\
22,275.02 &= 10,000(1+r)^5 \\
2.227502 &= (1+r)^5 \\
\sqrt[5]{2.227502} &= 1+r \\
1.173717609\cdots &= 1+r \\
0.173717609\cdots &= r
\end{aligned}$$

Normally in this book, we use 6 digits of precision, and therefore we'd report $17.3717\cdots\%$. However, for this problem, we're using 9 digits temporarily, and we report $17.3717609\cdots\%$. In industry, you would report a result normally to the nearest basis point, or 1% of 1%. That's 17.37%.



3-1-5

Now we have to compute the annual equivalent rate of Xavier's loan that compounds quarterly at $16\frac{1}{4}\%$, for a duration of five years.

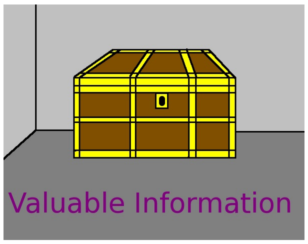
Again, we have two loans. The real loan is $16\frac{1}{4}\%$ compounded quarterly, while the hypothetical loan has unknown rate and is compounded annually. As before, the real and hypothetical loans go from the same P to the same A, in the same amount of time. Therefore, for both loans, we have $A = 22,176.09$ and $P = 10,000$, as well as $t = 5$.

$$\begin{aligned}
A &= P(1+i)^n \\
22,176.09 &= 10,000(1+r)^5 \\
2.217609 &= (1+r)^5 \\
\sqrt[5]{2.217609} &= 1+r \\
1.172673186\cdots &= 1+r \\
0.172673186\cdots &= r
\end{aligned}$$

Therefore, since we are temporarily using 9 digits of precision, we report $17.2673186\cdots\%$ instead of $17.2673\cdots\%$. In business, you will usually see interest rates reported to the basis point: 17.26% or 17.27%, depending on the circumstances.

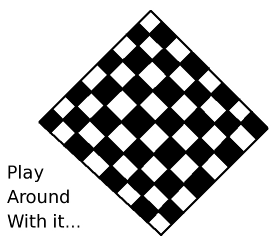
The above problem didn't have any fees. However, the whole point of this entire corner of mathematical finance is to aggregate all the costs of a loan together—from the fees, from the interest rate, and from the frequency of compounding—and describe this accurately with a single precise number.

Let's review the steps that we used, and try two practice problems without fees. Then we'll start in on how to analyze loans that include fees.



Here are the steps for how to compute the annual equivalent rate of a loan.

1. Compute the actual and final amount of the loan, which is often affected by fees.
2. Usually the principal is given, but sometimes the principal must be computed.
3. Consider a hypothetical loan that has the same A, the same P, and the same t.
4. Find the rate of the hypothetical loan. That r is the AER/EFF/EAR/APY/CAGR of the original loan.



3-1-6

Let us imagine that Quincy has a short-term loan of 2 years, arranged with a credit union. The rate is 4.8% compounded quarterly, and the principal is \$ 4000. This loan has no fees.

- Using the ordinary rules of compound interest, what is the amount? [Answer: \$ 4400.52.]
- What interest rate, compounded annually, brings a principal of \$ 4000 to an amount of \$ 4400.52, in two years? [Answer: $4.88708\cdots\%$.]

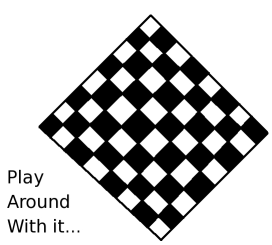
Therefore, we conclude that the AER is $4.88708\cdots\%$, which is normally written either 4.88% or 4.89%.

The way that we're going to check our work is that we're going to verify that 4.88708%, compounded annually, will really take a principal of $P = 4000$ to an amount of $A = 4400.52$.

This is just a classic, straight-forward compound interest problem.

$$A = P(1+i)^n = 4000(1+0.0488708)^2 = 4400.51982\cdots$$

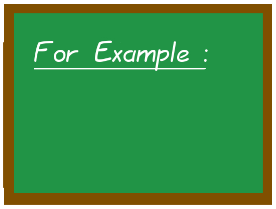
which is certainly close enough to the level of precision that we're using.



3-1-7

Repeat the previous checkerboard box using...

- ...6% compounded quarterly? [Answer: $6.13635\cdots\%$.]
- ...6% compounded monthly? [Answer: $6.16778\cdots\%$.]



3-1-8

Sometimes, loans have fees. Suppose you get a signature loan for \$ 5000, compounded quarterly for two years, at 15%. Suppose that there's a \$ 75 termination fee, a \$ 100 data-processing fee, and a \$ 50 records-disposition fee, each at the end of the loan. Consumers are very gullible, so they can be told that the fees at the end are to file the last bit of paperwork (which would take an underpaid bank employee a maximum of 15 minutes) and to clear all the records (which should be even faster). Let's try to analyze this loan, and compute a fair AER.

The amount A due at the end is going to be

$$(5000)\left(1+\frac{0.15}{4}\right)^{8} = 6712.35$$

but we need to add the fees.

You are paying $75+100+50 = 225$ extra from the fees. Therefore, when looking at the relative "quality" of this loan, we should consider the amount not as 6712.35, but rather

$$6712.35 + 225 = 6937.35$$

because that's what you're actually paying.

Now the question becomes, can we incorporate the idea of fees into the AER? We will do this in the next box.

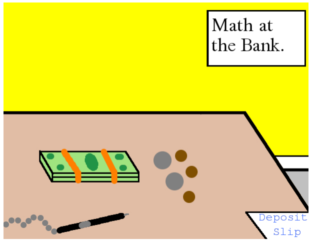
Continuing with the previous box, we are wondering if one can incorporate fees into the AER. The answer, it turns out, is yes—we can use the same concept. We are going to compute what rate of interest, compounded annually, on a hypothetical loan with no fees, with the same principle, and the same duration, would have the same amount.

$$\begin{aligned}
A &= P(1+i)^n \\
6937.35 &= (5000)(1+r)^2 \\
\frac{6937.35}{5000} &= (1+r)^2 \\
1.38747 &= (1+r)^2 \\
\sqrt{1.38747} &= 1+r \\
1.17790\cdots &= 1+r \\
0.177909\cdots &= r
\end{aligned}$$

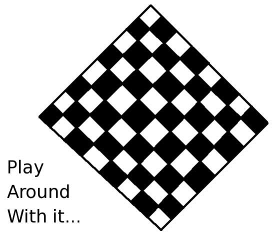
and so therefore, this loan is equivalent to an annually-compounded, no-fee loan at a rate of $17.7909\cdots\%$. That is considerably worse than the published rate of 15%. This rate is called an AER-with-fees.

Notice the following peculiarities of what we just did.

- The original loan was quarterly, but the hypothetical loan is annual. That's because we're computing an annual equivalent rate, not a quarterly equivalent rate nor a monthly equivalent rate.
- The original loan had fees, but the hypothetical loan does not have fees.
- Instead, the fees of the original loan, which were due at the end of the loan, are added on to the A of the original loan to make the A of the hypothetical loan.
- We had no fees at the start of the loan—we'll learn how to modify P to take those into account momentarily.



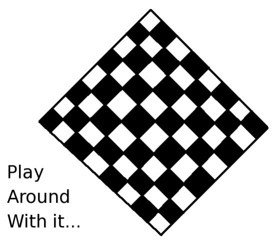
The *total interest paid* for a compound interest loan, just like a simple interest loan, is $A - P$. That's the amount you pay at the end, minus what you received at the beginning. If you add in the fees, this is called the *finance charge* of the loan.



3-1-9

Find the finance charge and the total interest paid in the previous example.

[Answer: $ 1937.35 was the finance charge and $ 1712.35 was the total interest paid.]



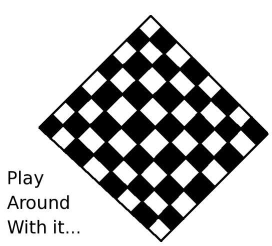
3-1-10

Go back to Xavier's loans on Page 405, and find the total interest paid, or TIP, for the three loans.

- What was the total interest paid, for the monthly loan? [Answer: $ 12,275.02.]
- What was the total interest paid, for the quarterly loan? [Answer: $ 12,176.09.]
- What was the total interest paid, for the annual loan? [Answer: $ 11,691.24.]

This problem tells us two things. First, it is good to be bank, and bad to be a borrower, based on those enormous TIP dollar amounts. Second, we confirm that the annual loan really is Xavier's best choice, because Xavier ends up paying the least amount—by a very significant margin, as you can see.

The exercise in the next box is a long one, but it is easily the most important checkerboard box in the entire module. It will take you a significant amount of time, but be sure to give it the attention that it needs.



3-1-11

Consider a loan with a principal of $ 10,000, and an interest rate of 10%. Let the loan be compounded monthly, and perhaps the fees total $ 400, payable at the end. For each of the following, find the AER-without-fees and the AER-with-fees:

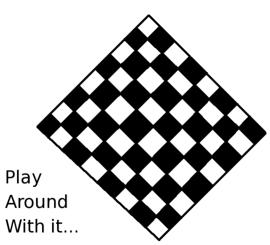
- If the loan is one year?
  [Answer: $10.4713\cdots$% without fees, and $14.4713\cdots$% with fees.]
- If the loan is two years?
  [Answer: $10.4713\cdots$% without fees, and $12.2671\cdots$% with fees.]
- If the loan is three years?
  [Answer: $10.4713\cdots$% without fees, and $11.5532\cdots$% with fees.]
- If the loan is four years?
  [Answer: $10.4713\cdots$% without fees, and $11.2056\cdots$% with fees.]
- If the loan is five years?
  [Answer: $10.4713\cdots$% without fees, and $11.0033\cdots$% with fees.]

Recall that compound interest becomes insanely powerful if either t or r becomes large. After doing the previous example, it should make sense that the AERs-with-fees slowly approach the AERs, either as t gets large or r gets large, because those fees matter less and less as you pay all that interest. However, if either t is small or r is small, then the fees might affect the AER-with-fees quite a bit.

Also, note that the AER-without-fees will never care about how long a loan is—but this is false for the AER-with-fees. For other uses of the AER, such as modeling stocks bought and sold with commission, the duration is important as well. We'll examine that case on Page 415.

Note, loans without fees (or more specifically, fees of \$ 0) always have

$$\text{AER-without-fees} = \text{AER-with-fees}$$



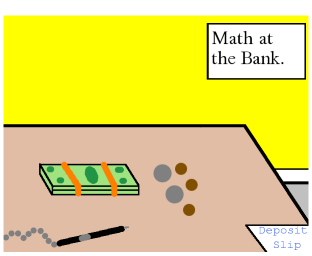
3-1-12

Suppose I can choose between two signature loans, for \$ 3000, offered to me by Bank Alpha and Bank Beta. For Bank Alpha, there is a loan termination fee of \$ 200, a records-disposition fee of \$ 100, and a service fee of \$ 75, with an interest rate of 12.75%, compounded monthly for a term of 2 years. (All those fees are due at the end of the loan.)

Alternatively, Bank Beta, is offering 15.75%, also compounded monthly for two years, with "no fees" written in big giant letters across the flyer.

- What are the AERs in each case? [Answer: $13.5221\cdots\%$ and $16.9381\cdots\%$.]
- What are the AERs-with-fees in each case? [Answer: $18.9002\cdots\%$ and $16.9381\cdots\%$.]
- Which bank is offering the better deal? [Answer: Bank Beta.]

As you can see, you save a lot of time on this type of problem if you remember that no-fee loans have an AER-with-fees equal to their AER-without-fees.



Do you see, now, why this topic is important? Normally, if you have a choice between a loan at $12\frac{3}{4}\%$ interest and a loan at $15\frac{3}{4}\%$ interest, you would think that you should take the loan with $12\frac{3}{4}\%$. This is particularly true because the fees are usually not written on the poster advertising the loan, except that sometimes you'll see "No Fees" written in huge letters.

You have to be savvy, and particularly remember to ask what the AER, EAR, or "effective rate of interest" is for the loan.

For Example :

3-1-13

Suppose Vito takes out a loan with an application fee of \$ 60, a processing fee of \$ 500, both due at the start, and loan-termination fee of \$ 250, due at the end. (You can already tell this is a bad loan, as the fees are quite high!) The loan is for \$ 24,000, for three years, and perhaps the interest rate is 5%, compounded monthly. What is the AER? and the AER-with-fees?

We should find the A first. The amount is

$$A = (24,000)(1+0.05/12)^{36} = (24,000)(1.00416\overline{6})^{36} = (24,000)(1.16147\cdots) = 27,875.33$$

Now we can compute the AER-without-fees, which is just written as "the AER," as follows:

$$\begin{aligned} A &= P(1+i)^n \\ 27,875.33 &= 24,000(1+r)^3 \\ \frac{27,875.33}{24,000} = 1.16147\cdots &= (1+r)^3 \\ \sqrt[3]{1.16147\cdots} = 1.05116\cdots &= 1+r \\ 0.0511618\cdots &= r \end{aligned}$$

Thus the AER is only $5.11618\cdots\%$.

We will compute the AER-with-fees in the next box.

Continuing with the previous box, we will now calculate the AER-with-fees. The fee at the end, of \$ 250, is added to the amount, which changes

$$27,875.33 + 250 = 28,125.33$$

and then the fees at the beginning are subtracted from the P. This might surprise you, and we'll discuss why this is done momentarily. For now, we have

$$24,000 - 60 - 500 = 23,440$$

and we can compute the AER based on this P and this A. We write

$$\begin{aligned} A &= P(1+i)^n \\ 28,125.33 &= 23,440(1+r)^3 \\ \frac{28,125.33}{23,440} &= (1+r)^3 \\ 1.19988\cdots &= (1+r)^3 \\ \sqrt[3]{1.19988\cdots} &= 1+r \\ 1.06262\cdots &= 1+r \\ 0.0626249\cdots &= r \end{aligned}$$

and we see that the AER-with-fees comes out to $6.26249\cdots\%$. In business, we'd normally report 6.26% or 6.27%. That's really different from the published rate of 5%.

Therefore, we can see that the fees can make a big difference to the total suffering that one must endure during a loan. Vito thinks he's getting a 5% rate but he's really getting 1–2 basis points over $6\frac{1}{2}\%$.

You're probably wondering why we subtract the fees at the start of the loan from P. I'll be explaining that momentarily. First, let's confirm that 6.26249% actually does take us from \$ 23,440 to \$ 28,125.33 in three years, compounded annually. That is straight forward:

$$P(1+i)^n = 23,440(1+0.0626249)^3 = 23,440(1.19988\cdots) = 28,125.3\cdots$$

This is excellent, and now we can be reassured that if our A and our P are correct, that we have correctly computed the AER-with-fees. With that in mind, we quickly verify

$$\begin{aligned} 23,440.00 + 60 + 500 &= 24,000.00 \qquad \leftarrow \text{YES!} \\ 28,125.33 - 250 &= 27,875.33 \qquad \leftarrow \text{YES!} \end{aligned}$$

In the previous boxes, since we kept the 6.26249% to six significant figures, we do not have any right at all, whatsoever, to expect the pennies to be correct in the number $28,125.3\cdots$. That's because the pennies would be the seventh significant figure. We would have to compute the AER, and all intermediate steps, to 8 or 9 significant figures if we wanted accuracy in the 7th significant figure of our final answer. Throughout this book, we use six significant figures, unless otherwise noted.

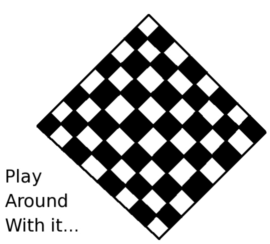
Now it is time to explain why we subtract fees payable at the start of the loan from the P when calculating the AER-with-fees. Reread the previous example, about Vito's \$ 24,000 loan. Suppose Vito goes into the bank to start the loan, and he has \$ 1000 on his person. First, he pays the \$ 60 fee, and has \$ 940. Then he needs to pay the \$ 500 fee, and has \$ 440. Now the banker gives Vito the principal of the loan, which is \$ 24,000. Finally, Vito leaves the bank, with \$ 24,440 on his person.

Since Vito entered the bank with \$ 1000, and left with \$ 24,440, then Vito has gained \$ 23,440. That's equivalent to a no-fee environment when Vito walks into the bank with \$ 1000, gets a $P = 23,440$ loan, and leaves with \$ 24,440.

That is why we compute

$$24,000 - 60 - 500 = 23,440$$

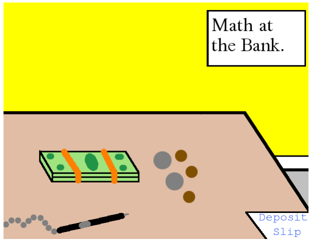
as the principal for Vito's loan. Of course, he's unlikely to pay the fees in cash, or carry that much cash with him on his person. Those details were just for the sake of visualization.



3-1-14

Suppose that an entrepreneur gets a 9-month small business loan for \$ 50,000, and the loan is compounded monthly, at a rate of 11.75%. The loan origination fee is 1% of the principal, payable at the start of the loan, and there are no other fees. Your job is to compute the AER? and the AER-with-fees? Hint: the duration of 9 months equals 0.75 years.

- What is the effective principal, after deducting the fees? [Answer: \$ 49,500.]
- What is the amount due at the end of the loan? [Answer: \$ 54,582.82.]
- What annual rate of interest will bring that principal to that amount? [Answer: $13.9203\cdots$%.]
- Now that we've computed an AER-with-fees of 13.9203%, compute the AER-without-fees. [Answer: $12.4039\cdots$%.]

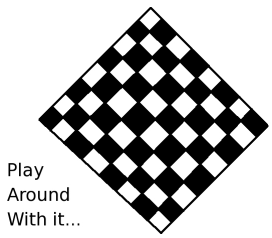


Let's look at the previous box again. Notice that both of these numbers (12.40% and 13.92%) are noticeably higher than the nominal rate, which was 11.75%.

If some entrepreneurs are offered either the same loan that Vito received, or alternatively a no-fee loan with a rate of 13.50% annually, many might choose Vito's loan because 11.75% is significantly lower rate than 13.50%.

However, that's a comparison of nominal rates. The correct way to compare loans is to use the AER, and in this case, because fees are present, we should use the AER-with-fees.

Because no-fee loan compounded annually at 13.50% has an AER-with-fees of exactly 13.50%, that would actually be a better option than Vito's loan with an AER-with-fees of 13.92%.

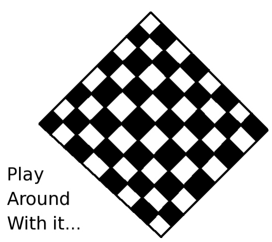


3-1-15

This is a true story. I received an envelope in the mail with an offer for a loan. I was very happy when I opened the envelope, because I saw immediately that it would make a fun math problem for this textbook.

The offer was for a cash transfer on my credit card, with interest at 28.99% compounded monthly. The fee was $ 15 plus 5% of the amount transferred. The fee was due immediately. I could transfer up to $ 12,000. If I were to transfer $ 12,000, then...

- What would the fee be? [Answer: $ 615.]
- What is the adjusted principal? [Answer: $ 11,385.]
- If I paid off the loan in one year, what would the amount due be? [Answer: $ 15,980.35.]
- What is the AER-with-fees? [Answer: $40.3631\cdots$%.]
- What is the AER-without-fees? [Answer: $33.1696\cdots$%.]

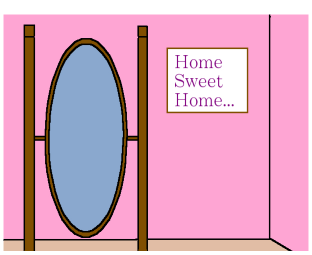


3-1-16

Now let's revisit the problem of the previous box, but imagine that I pay it back in 3 years.

- What would the amount due become, on that principle? [Answer: $ 28,339.81.]
- What is the AER-with-fees? [Answer: $35.5255\cdots$%.]
- What is the AER-without-fees? [Answer: $33.1696\cdots$%.]

These AERs are still horrible. The AER-without-fees is unchanged, but the AER-with-fees is smaller than in the previous box. With a longer loan, the fees "blend in" and are overshadowed by the compound interest.

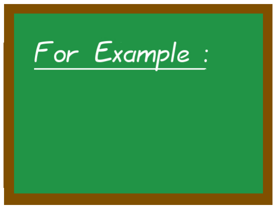


A Pause for Reflection...

Looking at the last two boxes, we have to ask ourselves: Isn't it horrible to pay $ 28,339.81 for just $ 12,000 worth of purchases?!

There are many ways in which lenders can become "predatory," inflicting great harm on borrowers. However, credit card companies often provide the most extreme examples.

The previous two boxes explain why you should never accept offers for loans that arrive, completely unsolicited, in your mailbox!



3-1-17

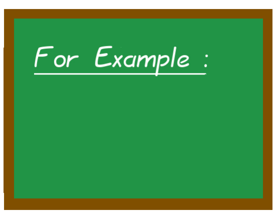
Suppose that I buy 75 shares of MathWorks stock at \$ 88.50 per share, and which I sell 3 years later for \$ 140.75 per share. The commission at purchase is \$ 66.38, and the commission at sale is \$ 105.56. How much do I pay for the first transaction? How much do I get back after the second transaction? What do the AER and AER-with-fees tell me?

First, the purchase is $75 \times 88.50 + 66.38 = 6703.88$. Then the sale is $75 \times 140.75 - 105.56 = 10,450.69$, and so the profit (which was not asked for, but which is interesting) is $10,450.69 - 6,703.88 = 3746.81$, not bad at all.

To find the AER-with-fees we will set $A = 10,450.69$ and $P = 6703.88$. Next, we have three years, and AERs (with or without fees) are computed as if they were compounding annually, so that's $n = 3$. We have

$$\begin{aligned}
A &= P(1+i)^n \\
10,450.69 &= 6703.88(1+r)^3 \\
\frac{10,450.69}{6703.88} &= (1+r)^3 \\
1.55890\cdots &= (1+r)^3 \\
\sqrt[3]{1.55890\cdots} = 1.15950\cdots &= 1+r \\
0.159505\cdots &= r
\end{aligned}$$

and so the AER-with-fees is 15.9505% or 15.95%, which is great. In the next box, we'll compute the AER-without-fees.

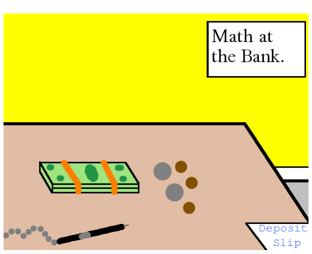


3-1-18

Continuing from the previous box, we've computed the AER-with-fees. Now to find the AER-without-fees, we repeat the problem without fees. First we would have $P = 75 \times 88.50 = 6637.50$ and $A = 75 \times 140.75 = 10,556.25$. Again, $n = 3$, and we write

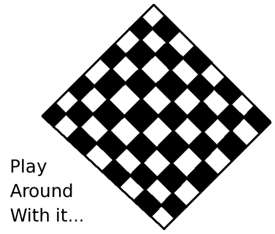
$$\begin{aligned}
A &= P(1+i)^n \\
10,556.25 &= 6637.50(1+r)^3 \\
\frac{10,556.25}{6637.50} &= (1+r)^3 \\
1.59039\cdots &= (1+r)^3 \\
\sqrt[3]{1.59039\cdots} = 1.16726\cdots &= 1+r \\
0.167262\cdots &= r
\end{aligned}$$

Thus the AER is 16.7262% or 16.72%.



Why was the AER higher than the AER-with-fees in the previous example, but in all the loan examples, it was lower? Well, consider the profit of the stock trades. It is $10,556.25 - 6637.50 = 3918.75$ without fees, and this is higher than the profit with fees, which is 3746.81. So the without-fees figure (AER) makes the investment look better (more profit). Likewise, the with-fees figure (AER-with-fees) makes the investment look worse (less profit). Because the AER makes the investment look better, it is higher than the AER-with-fees, which makes the investment look worse.

Alternatively, in an investment, the fees are fighting the interest. The interest is making you richer, while the fees are making you poorer. But in the case of a loan, the fees and the interest are cooperating to make you poorer. Therefore, in an investment, the with-fees figure (the AER-with-fees) should be bigger than the without-fees figure (the AER). You can think of the fees as an "interest-rate enhancing device" in the case of a loan, like a turbo-charger on a sports car's engine. For an investment, it is the exact opposite, it is an "interest-rate diminishing device," like a trailer-hitch attaching a boat to a car.

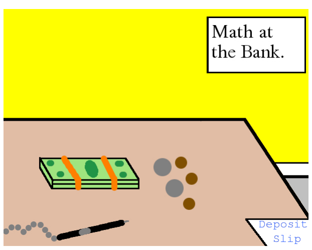


3-1-19

Suppose I buy 200 shares of Microsoft Corporation stock on April 25th, 2008, and sell them on April 25th, 2010. The price on April 25th, 2010 was \$ 30.96 per share and on April 25th, 2008, was \$ 27.34 per share. The commission at the purchase was \$ 60, and upon sale was \$ 65.

Hint: You might want to look at the previous example box as you set up this problem.

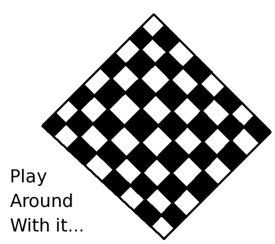
- How much were my shares worth at the start of this investment? [Answer: \$ 5468.00.]
- What did I pay at the start of this investment? [Answer: \$ 5528.00.]
- How much were my shares worth at the end of this investment? [Answer: \$ 6192.00.]
- What did I receive at the end? [Answer: \$ 6127.00.]
- What is the AER-without-fees of this investment? [Answer: $6.41460\cdots\%$.]
- What is the AER-with-fees of this investment? [Answer: $5.27855\cdots\%$.]



Normally, stocks out-perform bonds. Of course, that's obvious.

If stocks did not normally outperform bonds, i.e. if stocks were not normally more profitable than bonds, then no one would purchase stocks, since stocks are so much riskier than bonds. (We studied these matters in Module 2 "Intro to Portfolio Balancing" on Page 292.)

Yet as you can see from the previous problem, stocks do not *always* out-perform bonds, particularly during recessions. Many bonds can out-perform 5.28%, and with negligible risk.

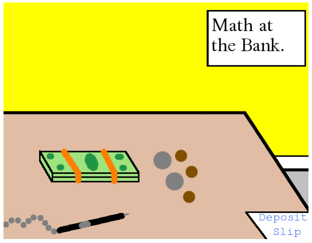


3-1-20

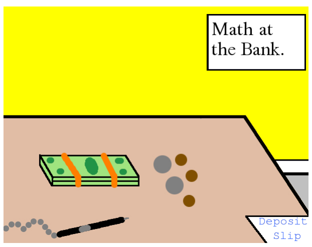
Suppose that, right after the market crash of 2008, Alice buys 100 shares of IBM stock on Nov 25th, 2008, for \$ 79.65 per share, and sells them on April 25th, 2010, for \$ 129.99 per share. The commission at purchase was \$ 80 and at sale was \$ 130.

- What did Alice pay at the start of this investment? [Answer: \$ 8045.]
- What did Alice receive at the end? [Answer: \$ 12,869.]
- How long is this investment? [Answer: 17 months or 17/12 years.]
- To the nearest basis point, what is the AER-without-fees of this investment? [Answer: 39.31%.]
- To the nearest basis point, what is the AER-with-fees of this investment? [Answer: 41.30%.]

As you can see, that is a phenomenally high rate of return. The moral of the story is that if you have spare cash available at the end of a stock-market plunge, it is a great idea to throw your extra cash into stocks. This is particularly true of large, famous, trustworthy corporations.

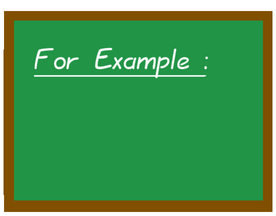


Given the above information, which should you use to judge an investment? And which should you use to judge a loan? The answer is that you should always use the AER-with-fees, because you do actually have to pay the fees! Since the fees have to be paid, the fees should be considered. By the way, on investments, usually one says APY (*annual percentage yield*) or CAGR (*compound annually growth rate*), which are synonyms for AER, and can either include or exclude fees as needed.



The most stupid possible indicator, and ironically the one most often used by business executives, is the *ROI* or "*Return on Investment*." This is the final profit divided by the total investment cost. Let's re-examine the MathWorks stock problem from Page 415.

The post-fees profit is 3746.81, and total investment cost (the principal with fees) is 6703.88. This comes to 55.89%. But did that happen in one year? In two years? In three years? Or in ten years? In this case, of course, it was 3 years. However, an investment instrument which achieved that profit-to-investment ratio over 20 years would be pathetic. We will examine that circumstance in more detail, in the next box.



3-1-21

To understand the comment at the end of the previous box, consider a 2.50% certificate of deposit for 20 years, compounded annually, and principal of 6703.88. What is the amount at the end, and what is the ROI?

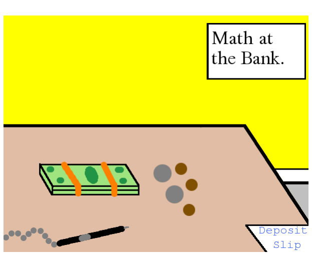
The amount is

$$A = 6703.88(1.025)^{20} = 10,985.08$$

and so the profit is $10,985.08 - 6703.88 = 4281.20$. The ROI of the certificate of deposit is $4281.21/6703.88 = 63.86\%$.

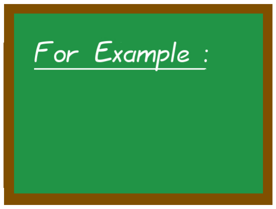
Thus the ROI in the previous paragraph of 55.89% is totally out performed by a measly certificate of deposit, not even a savings bond!

As you can see, the ROI is not a useful tool for comparing investments.



In conclusion, since the ROI ignores time, it is a foolish way to compare two investments, and is highly misleading. By the words "the ROI ignores time," I mean that no t or equivalent value, like n, appears in its formula.

The use of ROIs should be avoided, except when a customer asks for it.



3-1-22

Suppose your trading house uses the following system: all sales are subject to a commission of \$ 9.99 plus 0.5% of the value of the order, rounded up to the next penny. You purchase 19 shares of Miscbank at 9.89 a share on January 20th. You sell them for 9.97 a share on April 20th. What is the total paid at the start and end of the investment? What is the AER? And the AER-with-fees?

The purchase value is $19 \times 9.89 = 187.91$. The commission at buy would be $9.99 + 0.005(187.91) = 10.93$. The sale value is $19 \times 9.97 = 189.43$, and the commission at sale would be $9.99 + 0.005(189.43) = 10.94$. The amount paid at the purchase of the stock is $187.91 + 10.93 = 198.84$. The amount received at the sale of the stock is $189.43 - 10.94 = 178.49$, thus a definite loss.

For the AER we get $A = 189.43$ and $P = 187.91$. Note, April 20th is exactly 3 months after January 20th, or 1/4th of a year. Then we have

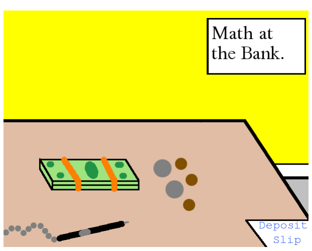
Without Fees

$$\begin{aligned} A &= P(1+i)^n \\ 189.43 &= 187.91(1+r)^{1/4} \\ \frac{189.43}{187.91} &= (1+r)^{1/4} \\ 1.00808\cdots &= (1+r)^{1/4} \\ (1.00808\cdots)^4 &= 1+r \\ (1.00808\cdots)^4 &= 1+r \\ 1.03275\cdots &= 1+r \\ 0.0327506\cdots &= r \end{aligned}$$

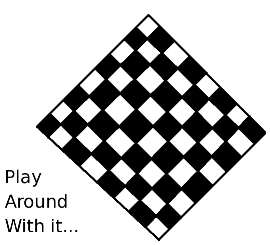
With Fees

$$\begin{aligned} A &= P(1+i)^n \\ 178.49 &= 198.84(1+r)^{1/4} \\ \frac{178.49}{198.84} &= (1+r)^{1/4} \\ 0.897656\cdots &= (1+r)^{1/4} \\ (0.897656\cdots)^4 &= 1+r \\ (0.897656\cdots)^4 &= 1+r \\ 0.649292\cdots &= 1+r \\ -0.350707\cdots &= r \end{aligned}$$

Finally, we can conclude that the AER is +3.27% while the AER-with-fees is -35.07%.



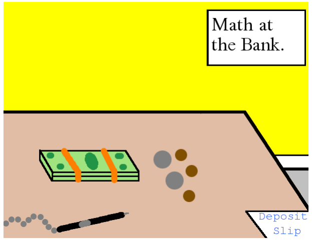
The previous example is a very good way to explain when you should use the AER versus the AER-with-fees. As you can see, the AER-with-fees is a more realistic model of what happened to the investor, who lost money. Yet the AER is a more realistic model of what happened to Miscbank, whose stock price was essentially unchanged but slightly higher. If you wish to talk about the company, you should use the AER. If you wish to talk about how the investment performs for the investor, you should use the AER-with-fees.



3-1-23

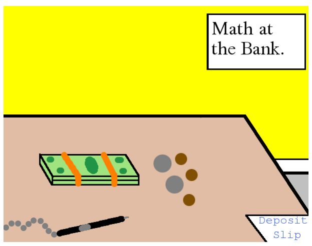
Repeat the above problem with General Electric, which went from \$ 16.40 on January 20th, 2010, to \$ 19.00 on April 20th, 2010.

- What is the amount paid at purchase? [Answer: \$ 323.14.]
- What is the amount received at sale? [Answer: \$ 349.20.]
- What is the AER-without-fees? [Answer: 80.15%.]
- What is the AER-with-fees? [Answer: 36.37%.]



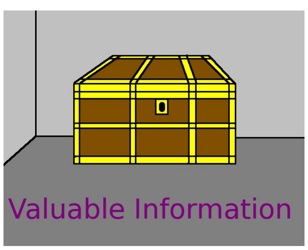
What we have been calling r is called, among many other names, the APR. The *Truth in Lending Act* is a Federal Law in the USA passed in 1968. It is from here that the method of calculation of the APR was codified into the law. Banks are required to compute the APR in this way, and to make it known to the customer in the loan paperwork. This is why car commercials often contain a statement of the APR, as do advertisements for loans in financial magazines.

Another rule is that investments (such as savings accounts) must disclose the APY, (annual percentage yield), which is another name for the AER. We'll continue the discussion in the next box.

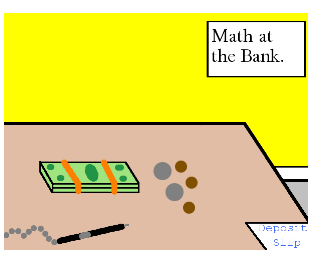


There are many other interesting facts about the *Truth in Lending Act*. An interesting factoid is the disclosure of the finance charge, which is made mandatory for all mortgages. This can be a shockingly high number. Remember, the *finance charge* is the sum of the *total interest paid* $(A - P)$ plus all the fees. We'll learn more about that in Chapter 4.

Of course, the law had many other effects, most of which dealt with the disclosure of information. Officially the law is called "Title 15, Chapter 41, Subchapter I, Part A, Sections 1601 to 1608" but most people call it the "Truth in Lending Act," or "TILA." Another name is "Regulation Z." If you want to read the act for yourself, Section 1606 defines the APR.



While EFF, EAR, CAGR, and APY are synonyms for the AER, it is important to remember that the APR is not a synonym for the AER. Instead, the APR is the ordinary r that we've been using throughout this textbook. Sometimes that r is called the published rate, or the nominal rate.



Recently, some legislation ("The Credit Card Act of 2009") was passed to limit the total application fees and various opening fees for a credit card, to be at most 25% of the maximum credit line. In any case, these types of cards are for people with very bad credit, who are looking to re-establish a credit rating after declaring bankruptcy.

There were many other provisions of the act, of course. For example, credit card companies are no longer able to give credit cards to college freshmen in return for a free t-shirt or free slice of pizza. In 1995, when I was a freshmen, I saw both of those happen (and I still have the t-shirt).

Let us consider how credit cards can use fees to create profit through the following example, which will also show how AERs can expose that scam.

For Example :

\# 3-1-24

Consider a card with a max of \$ 1000, a \$ 150 application fee, and a \$ 100 account-opening fee. Suppose the interest rate is not so bad, perhaps 19.95%. (Yes, that's very high, but credit cards often have rates similar to that.) Suppose someone buys something worth \$ 1000, and pays everything off after one month. (Just to be clear, that means one month after the purchase, they make a single payment for the purchase, any interest, and the fees). How much have they paid, and what is the AER-without-fees? and the AER-with-fees? Note that the fees will be going on the card's balance, and therefore will be paid off when the card is paid off—at the end of the loan.

Well, they have paid \$ 250 in fees, plus one month of interest. If $r = 0.1995$, then $i = 0.1995/12 = 0.016625$. Next, we can calculate $A = (1000)(1 + 0.016625)^1 = 1016.62$. Thus we see \$ 16.62 in interest, \$ 1000 in principle, and \$ 250 in fees.

To find the AER without fees, we set $A = 1016.62$, to write

$$\begin{aligned} 1016.62 &= (1000)(1+r)^{1/12} \\ 1.01662 &= (1+r)^{1/12} \\ 1.01662^{12} = 1.21871\cdots &= (1+r) \\ 0.218719\cdots &= r \end{aligned}$$

but to find the AER with fees, we add \$ 250 to A and thus set $A = 1266.62$, to write

$$\begin{aligned} 1266.62 &= (1000)(1+r)^{1/12} \\ 1.26662 &= (1+r)^{1/12} \\ 1.26662^{12} = 17.0512\cdots &= (1+r) \\ 16.0512\cdots &= r \end{aligned}$$

and thus we conclude that the AER-with-fees is 1605.12% while the AER is 21.8719%. Is this case pathological? Yes, it is. This consumer is being taken advantage of, and this normally should not happen. After all, the consumer is paying \$ 1266.62 for only \$ 1000 worth of stuff!

Play Around With it...

\# 3-1-25

Take the above example, but let the credit line be \$ 2000 instead of \$ 1000. Assume again that the owner of the card goes on a shopping-spree on the first day, maxing out the card, and pays it off after one month.

- How much does the consumer pay in total? [Answer: \$ 2283.25.]
- What is the AER-without-fees? [Answer: $21.8863\cdots$%.]
- What is the AER-with-fees? [Answer: $390.096\cdots$%.]

Play Around With it...

\# 3-1-26

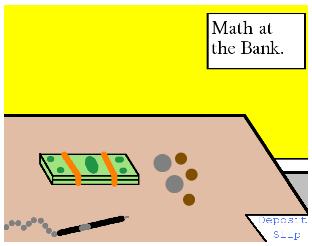
Repeat the previous checkerboard box with \$ 500 for purchases, not \$ 1000 or \$ 2000.

- How much does the consumer pay in total? [Answer: \$ 758.31.]
- What is the AER-without-fees? [Answer: $21.8863\cdots$%.]
- What is the AER-with-fees? [Answer: $14,708.8\cdots$%.]

That last AER-with-fees in the previous box is so ridiculous, that we really do need to check our work to make sure it makes sense!

- If the AER-with-fees is $14{,}708.8\cdots\%$, then $r = 147.088\cdots$, and since AERs are annual, $m = 1$ thus $i = r/m = 147.088\cdots$. Next, $t = 1/12$ and
$$n = mt = (1)(1/12) = 1/12$$
Note, $P = 500$ because that was the total of the purchases. Then,
$$A = P(1+i)^n = (500)(1+147.088\cdots)^{1/12} = (500)(1.51662\cdots) = 758.309\cdots$$
which matches to the nearest penny.

- While we are at it, let's check the box above that previous box. If the AER-with-fees is 390.096%, then $r = 3.90096$. The principal is \$ 2000, and since $m = 1$, then $i = r/m = 3.90096$, and $n = 1/12$. Then,
$$A = P(1+i)^n = (2000)(1+3.90096)^{1/12} = (2000)(1.14162\cdots) = 2283.24\cdots$$
which is a match, off only by a fraction of a penny.



Many credit cards give a 30-day grace period, where new purchases do not charge you in interest until they are 30-days old. This way, if you pay the card off each month, then you never pay any interest. People with bad credit are not offered these sorts of cards, so we rightfully assumed that this was not the case in the above examples, and that purchases charged interest from the moment they were made. However, most cards use the grace period, especially those issued to professionals with good financial records.

You should consider this detail seriously when choosing credit cards—do not take a credit card without the grace period, unless you are in serious financial distress.

We're about to derive a shortcut formula that some people use when computing an AER-without-fees for an interest rate. The derivation of the shortcut formula might look intimidating. However, you do not have to understand why a formula is true in order to use the formula. Therefore, do not panic if this derivation is too complex or symbol-heavy to be easily read. On the other hand, I earnestly believe that if you try to have an understanding of the derivation, then you'll be advantaged from that investment of time, because you will be less likely to become confused and make errors.

We're going to begin our derivation by surveying the "alphabet soup" that comes up in compound interest. As before, we consider a real loan and a hypothetical loan. In the absence of fees, the A and the P are the same for the real and the hypothetical loan. The time t, or duration, is also unchanged. The r will be different for the two loans—for the real loan it is the nominal rate or APR, and for the hypothetical loan it will be the AER, EAR, EFF, or APY, depending on which abbreviation you prefer. We will continue in the next box.

Likewise m will be something important for the real loan, depending on whether the real loan is quarterly, monthly, biweekly and so forth. Contrastingly, the m is always 1 for the hypothetical loan, because the AER is an annual effective rate—not a quarterly effective rate or a monthly effective rate. Because m is different, that means n and i will be different as well.

We can mark with a subscript of r the variables for the real loan, so that r_r, n_r, i_r, and m_r, represent r, n, i and m for the real, and for the hypothetical we write r_h, n_h, i_h, and m_h. The derivation continues in the next box.

Because $A_r = A_h$, then we should have the formula

$$P\left(1+i_r\right)^{n_r} = P\left(1+i_h\right)^{n_h}$$

which comes from the fact that the amount in the real loan and in the hypothetical loan should be identical. We can see right away that we can cancel the Ps. So then we'd have

$$\left(1+i_r\right)^{n_r} = \left(1+i_h\right)^{n_h}$$

but we can plug in $i_r = r_r/m_r$, and also $n_r = m_r t$. Now we've got

$$\left(1+\frac{r_r}{m_r}\right)^{m_r t} = \left(1+i_h\right)^{n_h}$$

Similarly, we could plug in $i_h = r_h/m_h$ and $n_h = m_h t$. However, we should remember that the hypothetical loan is annual, and thus $m_h = 1$. Therefore $i_h = r_h$, and $n_h = t$. This brings us to

$$\left(1+\frac{r_r}{m_r}\right)^{m_r t} = \left(1+r_h\right)^{t}$$

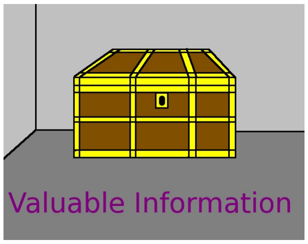
Taking the "t"th root of both sides brings us to

$$\left(1+\frac{r_r}{m_r}\right)^{m_r} = 1+r_h$$

and we subtract one from each side to get

$$\left(1+\frac{r_r}{m_r}\right)^{m_r} - 1 = r_h$$

The derivation is now complete, because we have isolated r_h, the rate of the hypothetical loan, which is what we wanted. That rate will be the AER, EAR, EFF, or APY, depending on which abbreviation you prefer.



For a loan without fees, the AER can be computed by

$$EFF = AER = EAR = APY = \left(1+\frac{r}{m}\right)^{m} - 1$$

However, this formula does not work computing the AER-with-fees for a loan or investment that carries fees.

Personally, I do not recommend memorizing the above formula. The techniques of this module always work, with or without fees. Therefore, it is silly to go through the effort of learning a new formula, solely for the circumstance of the fees equaling zero. In any case, the formula is frequently given in competing textbooks, so we should probably explore it just for a few boxes before moving on.

For Example :

3-1-27

Suppose you have the choice of four small business loans. The first has the rate at 9.5%, and is compounded monthly. The second has the rate at 9.75%, and is compounded quarterly. Third is at 10%, and is compounded semiannually. The fourth is at 10.25%, and is compounded annually.

Let us calculate the AER in each case, using the shortcut formula. First, we'll calculate the monthly value.

$$\text{Monthly: } \left(1+\frac{0.095}{12}\right)^{12} - 1 = 0.0992475\cdots$$

Continuing from the previous box, we can similarly calculate the other values. We have

$$\begin{aligned}
\text{Quarterly: } \left(1+\frac{0.0975}{4}\right)^{4} - 1 &= 0.101123\cdots \\
\text{Semiannually: } \left(1+\frac{0.1}{2}\right)^{2} - 1 &= 0.1025 \\
\text{Annually: } \left(1+\frac{0.1025}{1}\right)^{1} - 1 &= 0.1025
\end{aligned}$$

and we can see that the monthly loan (i.e. the first loan) is the best choice, because it has the lowest AER.

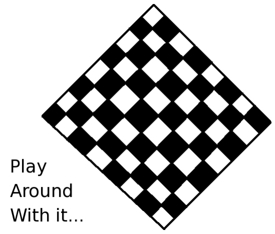
Play Around With it...

3-1-28

Instead of the above, consider how things would change if the interest rates were what is given below. Find the AERs, and use six significant digits for practice.

- 19% compounded monthly? [Answer: $20.7450\cdots$%.]
- 19.25% compounded quarterly? [Answer: $20.6847\cdots$%.]
- 19.50% compounded semiannually? [Answer: $20.4506\cdots$%.]
- 19.75% compounded annually? [Answer: 19.75%.]

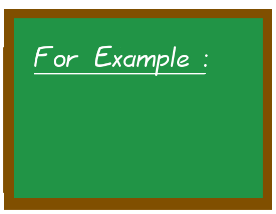
Note that the AER is equal to the nominal rate for any no-fee, annually compounded, loan or investment.



3-1-29

Interbank loans often have extremely low rates. Consider four possible loans of one year, listed below, and find the AER. Remember, this book uses a 360-day year. Because these numbers are small, be sure to give six significant figures.

- A rate of 1.25%, compounded daily. [Answer: 1.25782%.]
- A rate of 1.50%, compounded weekly. [Answer: 1.51108%.]
- The next bullet is going to use the word "bimonthly," which in this textbook, and most circumstances in industry, means "6 times per year" or "every other month." However, sometimes in industry the word "bimonthly" means "24 times per year" or "twice per month."
- A rate of 1.75%, compounded bimonthly. [Answer: 1.76281%.]
- A rate of 2.00%, compounded monthly. [Answer: 2.01843%.]



3-1-30

Let us suppose that a member of your staff is shopping around for signature loans, to help you start a spin-off business. The first loan found is 14.5% compounded quarterly. You can calculate that the AER is $15.3076\cdots\%$ but your staff member does not know how to do this. There are a lot of loan offerings out there, including annually, quarterly, and monthly. You don't want to calculate the AER for all of them. Obviously a quarterly loan that beats 14.5% is a better loan, or an annual loan that beats 15.3076%, because that is what the AER means. What would the monthly loans have to have as a rate in order to be better?

$$\begin{aligned} (1+r/12)^{12} &= 1.153076\cdots \\ \sqrt[12]{(1+r/12)^{12}} &= \sqrt[12]{1.153076\cdots} \\ 1+r/12 &= 1.01194\cdots \\ r/12 &= 0.0119401\cdots \\ r &= 0.143282\cdots \end{aligned}$$

Therefore, we learn that our loan is equivalent to 14.32% compounded monthly, and the staff member can search the loan offerings on that basis. For example, a loan that is 14.40% compounded monthly, even though it has a lower nominal rate, has a higher AER and so should be rejected.

A great way to check your work in the previous box is to plug the 14.3282% into the AER formula and find its AER.

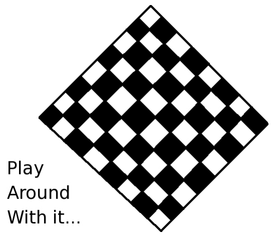
$$\left(1+\frac{0.143282}{12}\right)^{12} - 1 = 0.153076\cdots$$

which is exact to six decimal places.

Yet, you could also check that a 14.40% compounded monthly loan has a higher AER than the 14.50% compounded quarterly that we've already found.

$$\left(1+\frac{0.144}{12}\right)^{12} - 1 = 0.153894\cdots$$

and yes, that is larger than 1.153076, therefore all is well.

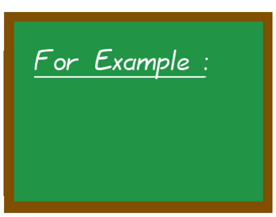


3-1-31

Rarely would a signature loan be available on a weekly or daily basis, but it is theoretically possible.

- What would the nominal rate of a weekly compounded loan, that has the same AER as the loan in the previous example box? [Answer: 14.2628%.]
- What would the nominal rate of a daily compounded loan, that has the same AER as the loan in the previous example box? (Use a 360-day year.) [Answer: 14.2461%.]

So as you can see, the AER is kind of a "Grand Central Station" for loan rates. You can switch from monthly to weekly or daily to annually by using the AER, just as you would change train lines by going to Grand Central.

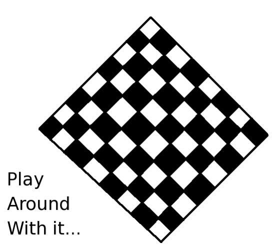


3-1-32

Once in a rare while, we might want to convert an AER (without fees) back into its nominal rate. That's the original publish rate, that we've been calling r. For example, if we know that an investment which is compounded quarterly has an effective rate of 7.97815%, we might want to know what the nominal rate is. We can just use the shortcut formulas for that.

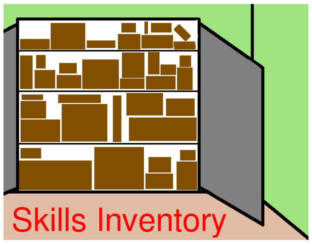
$$\begin{aligned}
\left(1+\frac{r}{m}\right)^m - 1 &= AER \\
\left(1+\frac{r}{4}\right)^4 - 1 &= 0.0797815 \\
\left(1+\frac{r}{4}\right)^4 &= 1.07978\cdots \\
\sqrt[4]{\left(1+\frac{r}{4}\right)^4} &= \sqrt[4]{1.07978\cdots} \\
1 + r/4 &= 1.01937\cdots \\
r/4 &= 0.0193749\cdots \\
r &= 4(0.0193749\cdots) = 0.0774999\cdots
\end{aligned}$$

As you can see, that's clearly 7.75% which is $7\frac{3}{4}$%.



3-1-33

Perhaps the previous example dealt with a stock fund offered by a big brokerage house. If they also have a bond fund that returns quarterly, we might want to know its nominal rate. Suppose the effective rate is 4.83528%, then what is the nominal rate?
[Answer: $4.74999\cdots$% which is clearly $4\frac{3}{4}$%.]



We have learned the following skills in this module:

- To realize that you cannot compare loans merely by comparing the interest rates,
- To calculate the AER of a loan,
- To calculate the impact of fees on the AER,
- To calculate the total interest paid, and the finance charge,
- To apply the AER to case of stock trades with commissions,
- To compute the Return on Investment or ROI, and why it should not be computed,
- To derive and use the shortcut formula for an AER-without-fees,
- To convert among different frequencies of compounding by using the AER as "Grand Central Station,"
- To calculate what nominal rate would produce a given AER,
- As well as the vocabulary terms: AER, Annual Effective Rate, Annual Equivalent Rate, Annual Percentage Yield, APY, CAGR, Compounded Annually Growth Rate, EAR, EFF, Effective Annual Rate, Effective Rate of Interest, Finance Charge, ROI, Return on Investment, Signature loans, Total Interest Paid.