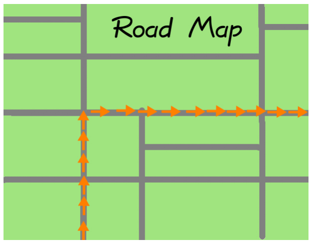
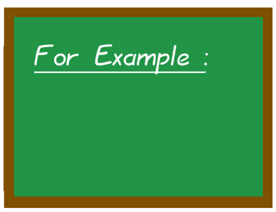
Module 3.4: Supply and Demand: Part One



The Theory of Supply & Demand is among the most famous of all theories, and easily the most famous in economics. Here, we will be exploring that theory in a slightly more precise way than most students will have seen before. The problems will not be difficult, and we'll get a more concrete understanding of the theory than would be possible by using mere words and skipping the mathematics.



3-4-1

Let's imagine that you are working in the realty-management office of a vacation rentals facility in a nice but affordable resort area. There is a large condominium of essentially identical vacation apartments. Using the techniques that will be taught throughout this module and the next, it might be the case that the demand for vacation rentals is given by

$$d(p) = -p + 320$$

where p is the price per week of the average rental, and $d(p)$ is the number of units that get rented. Meanwhile the supply is given by

$$s(p) = p - 50$$

Now compute what the supply and demand will be if, in one season, the going rate is $p = 120$.

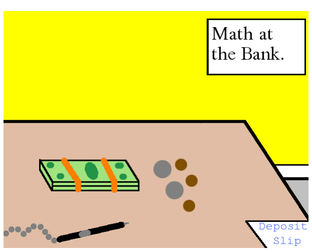
We have for demand

$$d(120) = -120 + 320 = 200$$

while for supply

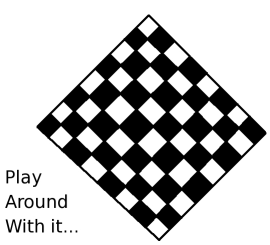
$$s(120) = 120 - 50 = 70$$

and therefore there is a demand for 200 units, but a supply of 70 units. The official term for when the demand exceeds the supply in economics is a *shortage*.



In the above circumstance, what will happen? Since the demand exceeds the supply, some customers will be disappointed. Initially, landlords will react by phasing out discounts and other "perks" meant to attract renters. Next, a few savvy landlords will begin to raise their prices. Also, some disappointed potential renters might try to secure a rental by "overbidding," which means they voluntarily try to pay more, to take the place of a renter unwilling to overbid.

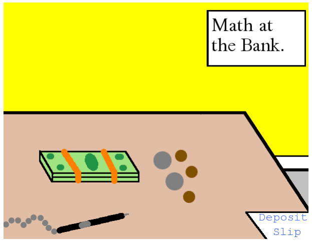
In summary, during a shortage, the price will go up.



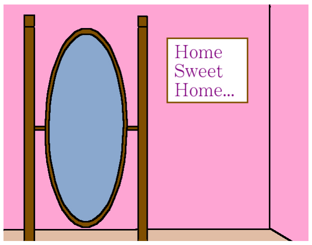
3-4-2

Continuing with the previous example, perhaps the landlords will overreact to the shortage in one season by raising prices too much for the next season. Compute the supply and demand if the average price is raised to \$ 220 per week.

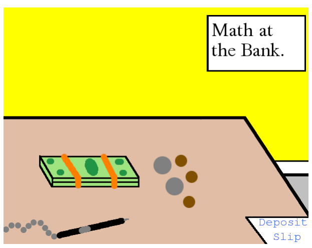
- What is the demand? [Answer: 100 rental units.]
- What is the supply? [Answer: 170 rental units.]



In this situation, where the supply exceeds the demand, we use the word *surplus*. What will happen? Suppliers might offer discounts to encourage sales, or toss in some perks. Customers will shop around and other suppliers must offer similar discounts—or outright lower their price. Savvy renters will play landlords against each other, making counteroffers to get a lower price. Maybe some stubborn landlords will not lower their price and might discover their property is vacant during several weekends. Then they will offer bargain prices to at least get some revenue of some kind for those weekends.



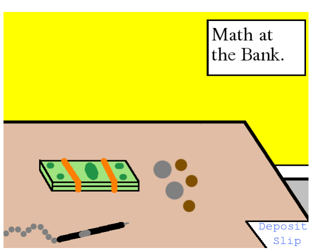
A Pause for Reflection...
By this point, we are probably convinced that during a shortage, the price will go up, and that during a surplus, the price will go down. However, can these processes continue forever? In a shortage, will the price continue to rise and rise? During a surplus, will the price continue to fall and fall? Let's think about this in some detail now.



During a shortage, the price cannot go up forever. Some potential renters will be priced out of the market—discovering that they simply cannot afford a vacation rental. Perhaps a few frequent renters will choose to rent less often, because of the increasing price. Other renters might decide to rent in a different part of the country, where it might be cheaper. In other words, as the price continues to rise, the demand will decrease.

Also some landlords will buy a second property, perhaps rent out their basement, or even move out of their own house into one that is a bit farther away, to rent their old house out. Some entrepreneurial landlords might even build a new duplex or quadraplex to rent out. In other words, as the price rises the supply will go up.

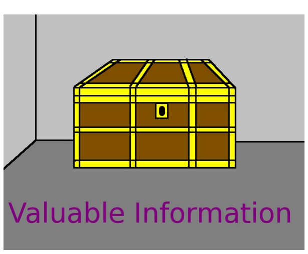
Both of these forces are pushing the market toward "a happy medium," called an *equilibrium*, where the supply and demand are equal.



Likewise, during a surplus, the price cannot go down forever. As the price decreases some landlords will decide that the profit, which would become smaller and smaller, is not worth the hassle. They might decide to stop being a landlord altogether, or perhaps to let an unemployed relative (such as a college student or an elderly mother-in-law) stay for free in the rental. Maybe they will decide to simplify their life by selling their rental property and just investing the money in stocks and bonds. In other words, as the price falls, the supply will decrease.

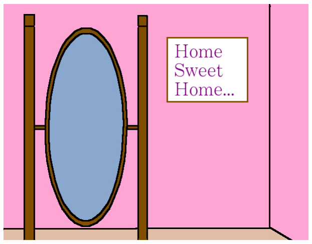
While all this is happening to the suppliers as the price decreases, occasional renters might start renting more frequently, or for longer time periods. Customers who previously imagined themselves unable to afford a vacation rental might now discover that they are able to. In other words, as the price decreases, the demand will go up.

As before, both of these forces are pushing the market toward "a happy medium" or equilibrium, where the supply and demand are equal.



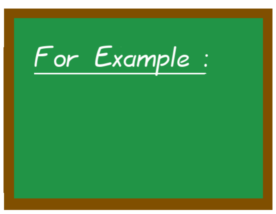
In grand summary of the above:

- In a shortage, the demand exceeds the supply; prices will go up; demand will go down; and the supply will go up.
- In a surplus, the supply exceeds the demand; prices will go down; demand will go up; and the supply will go down.



A Pause for Reflection. . .
Note that it is much wiser to think about these relationships whenever you need to recall one of these facts. You should *not attempt to memorize* these facts, because you might memorize them incorrectly. It is far better to think about the situation, perhaps imagining your favorite product (e.g. chocolate, beer, or video games) and then common sense will render everything remarkably clear.

For example, after Valentine's Day has ended, a corner store might have a remarkably large quantity of extra chocolate on hand. Are they going to raise their price or lower it? Of course, they will lower it! The demand for chocolate immediately after Valentine's Day is much less than the demand immediately before it. If they have excess supply, they should lower their prices to get that chocolate moving out the door. Therefore, you do not need to memorize "if the supply gets high, the price will drop" if you can just imagine chocolate.



3-4-3

Continuing with our previous example, Is there a price at which the supply and demand are equal? Let's find out!

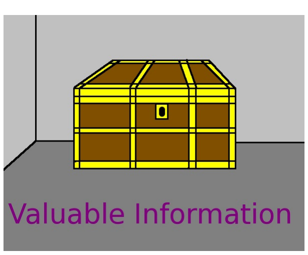
$$\begin{aligned} s(p) &= d(p) \\ p - 50 &= -p + 320 \\ 2p - 50 &= 320 \\ 2p &= 370 \\ p &= 185 \end{aligned}$$

It seems that at a price of \$ 185 per weekend, the market's supply and demand should be in balance. This is called the *equilibrium price*. However, we should check our work, which we will do in the next box.

In a supply and demand problem, it is easy to check your work by plugging the equilibrium price into both the supply function and the demand function, and seeing that they come out the same.

- $s(185) = 185 - 50 = 135$
- $d(185) = -185 + 320 = 135$

This is an exact match, and so we can be confident that we have successfully computed the equilibrium price.



The price at which the supply function and the demand function are equal is called the equilibrium price. At that price, both the supply and the demand come out to the same number, which can be called either the *equilibrium quantity*, *equilibrium supply*, or the *equilibrium demand*. (They are synonyms.)

Play Around With it...

3-4-4

Using techniques that we will learn very soon, perhaps the Department of Agriculture has learned that a particular type of soy has a nationwide demand function of

$$d(p) = -300p + 400 \text{(millions of pounds)}$$

while the supply function, again for the entire country, is given by

$$s(p) = 200p - 100 \text{(millions of pounds)}$$

(Note that in both functions, p is the price in dollars per pound.)

- What is the equilibrium price? [Answer: $ 1 per pound.]
- What is the equilibrium supply? [Answer: 100 million pounds.]
- What is the equilibrium demand? [Answer: 100 million pounds.]

As you can see from the previous box, the equilibrium supply and the equilibrium demand are the same. This will always be the case—and it is a favorite exam question of many instructors.

Play Around With it...

3-4-5

Referring to the previous checkerboard box...

- If the price in one season were 75 cents per pound, what would the demand be? [Answer: 175 million pounds.]
- If the price in one season were 75 cents per pound, what would the supply be? [Answer: 50 million pounds.]
- If the price in one season were $ 1.20 per pound, what would the demand be? [Answer: 40 million pounds.]
- If the price in one season were $ 1.20 per pound, what would the supply be? [Answer: 140 million pounds.]

An agricultural economist might refer to the above as a shortage of 125 million pounds (at 75 cents), and a surplus of 100 million pounds (at $ 1.20), both of which would be extreme situations.

Play Around With it...

3-4-6

Now we're going to imagine that you are an intern at a pharmaceutical company that makes one of many generic versions of a popular and famous allergy remedy. The economic-forecasting department tells you that their research indicates supply and demand functions of

$$d(p) = 903.8 - 0.1p \qquad \text{and} \qquad s(p) = 15p - 1180$$

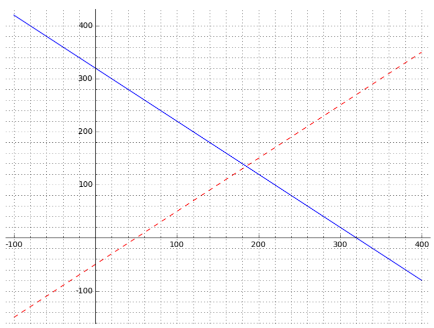
for the first-world markets, where p is in dollars per package and s and d are in thousands of packages.

- What is the equilibrium price? [Answer: $ 138 per package.]
- What is the equilibrium supply? [Answer: 890,000 packages.]
- What is the equilibrium demand? [Answer: 890,000 packages.]

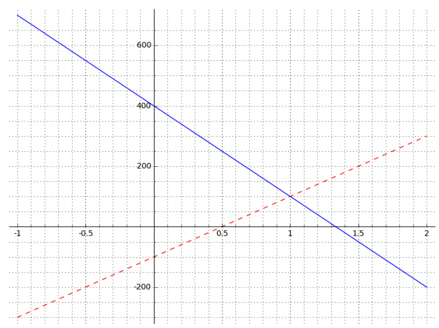
In the next box below are found the graphs of the supply and demand functions for the three examples we have considered so far. In each graph, the supply function is the red dashed line, and the demand function is the blue solid line.

A few things can be observed right away. The supply function is always increasing, and this will be true for all supply functions, including nonlinear ones. Likewise, the demand function is always decreasing, and this will be true even for nonlinear demand functions. The place where the lines cross is the equilibrium, and you can check to see that the coordinates of the point where they cross appear to match the values which we calculated so carefully.

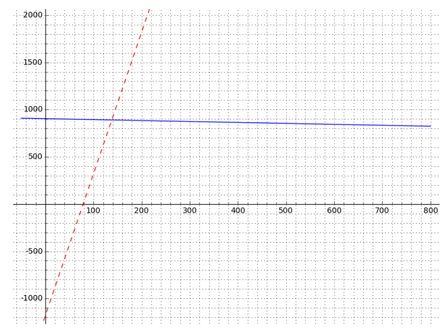
For example, in the leftmost graph, we see that the vacation-rental problem equilibrium falls between \$ 180 and \$ 200 (leaning toward \$ 180), and between 120 and 140 units, leaning toward 140. Our actual computed values were \$ 185 and 135 units.



Vacation Rentals
$d(p) = -p + 320$
$s(p) = p - 50$



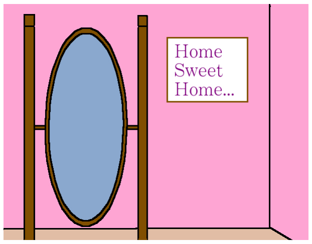
Soy
$d(p) = -300p + 400$
$s(p) = 200p - 100$



Allergy Pills
$d(p) = 903.8 - 0.1p$
$s(p) = 15p - 1180$

Looking at those graphs in the previous box again, the supply function enters Quadrant I on the x-axis, while the demand function enters Quadrant I on the y-axis. This fact might seem like a minor detail now, but it will become an important fact shortly, when we talk about the maximum feasible price and minimum feasible price.

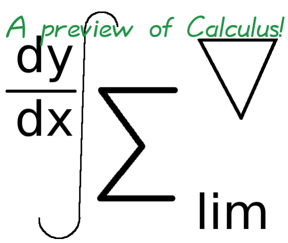
Some textbooks have the price as the y-axis, and the quantity (either the supply or the demand) as the x-axis. Other textbooks, including this one, have the price as the x axis and the demand as the y-axis. It is unfortunate that this inconsistency exists, but it probably won't impact you in any way. However, I feel as though I should warn you about it because you might see Supply & Demand questions in other courses, such as economics.



A Pause for Reflection...
Looking at the box before last, with those three graphs inside, did you notice the demand curve for the allergy pills has a very shallow slope, compared to the demand curves in the other two graphs? This is a key observation. Consider the following five values, using the function $d(p) = 903.8 - 0.1p$ we had earlier:

$$d(108) = 893 \quad d(148) = 889 \quad d(188) = 885 \quad d(228) = 881 \quad d(268) = 877$$

As you can see, as the price per package was increased by \$ 40 each time, the demand dropped by only 4, meaning four thousand packages. While the price of \$ 268 per package is significantly more than double the price of \$ 108 per package, the demand only fell from 893,000 packages to 877,000 packages—a difference of 16,000 packages. That's somewhat small compared to the number of packages being sold. Of the 893,000 people that would be willing to buy a package at the price of \$ 108, there are 877,000 who would be willing and able to pay \$ 268 per package—the other 16,000 people either cannot pay or are unwilling to pay.



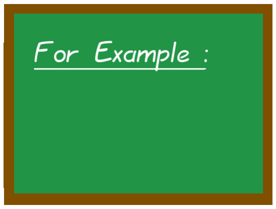
The phenomenon we observe here is very common in the market for consumer medical products. One can say that a product has *inelastic demand* if the demand does not change much with the price; alternatively, one can say that a product has *elastic demand* if the demand does change a lot with the price. Of course, these terms are descriptive but imprecise. The *elasticity of demand* is a precise measure, and one that we cannot define here. The computation of the elasticity of demand requires calculus, but there is a way to do it approximately using only algebra.

In any case, we would say that life-saving medicine has highly inelastic demand, because people are willing to pay a lot of money to avoid death. Non-life-saving medicines are still inelastic, but less so.

A fascinating paper was published in 1994 by the Oxford journal *Health Policy and Planning*, by Rainer Sauerborn, Adrien Nougtara and Eric Latimer. Their paper, titled "The Elasticity of Demand for Health Care in Burkina Faso: Differences Across Age and Income Groups," precisely measured the values of the elasticity of demand for many groups, including health care for the poor, for infants, and for children.

Since Burkina Faso is one of the poorest countries in the world, this is extremely informative, revealing dark corners of the economic decision-making mechanisms of the mind. How do the poorest of the poor decide whether or not to provide healthcare to their infants or their children, when they can scarcely afford it? This is exacerbated by the fact that families in Burkina Faso tend to be large—an average of six children will be born to each woman according to the CIA's *World Factbook*.

To withhold life-saving health care from one's own children at first sounds unthinkable, but when faced with the possibility of starving an entire family, and thus losing several children, a parent might indeed choose to save money by withholding healthcare to be able to buy food—thereby sacrificing the life of only one infant child rather than losing the entire family. The paper is a bit advanced, but after a few courses in economics or *Business Calculus* it should be comprehensible.



Commodity prices are often the targets of Supply & Demand analysis, since the data is relatively available and carefully compiled. Now we're going to look at a long example over several boxes that has to do with silicon. Suppose that Alice has an internship with a company that makes computer chips. It might become very useful to make a prediction of where the price of silicon is going. Accordingly, Alice contacts the research department, to get some information about the world total production of silicon.

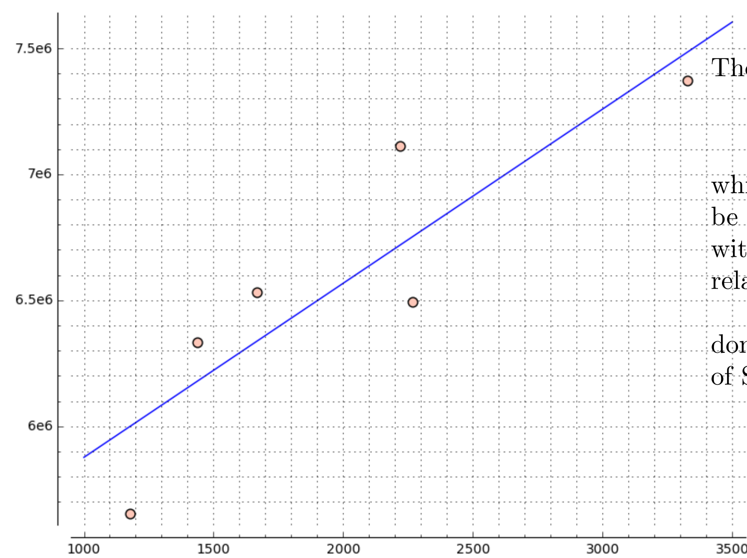
In the next box will be the raw data that Alice got from the research department, along with some of her computations.

3-4-7

Continuing with the previous box, here is the data which Alice obtained

Year	Price	Actual Production	Predicted Production	Residual Error	Relative Error
2006	1180	5,650,000	6,000,489	350,489	6.20335%
2007	1440	6,330,000	6,180,218	-149,781	-2.36621%
2008	2270	6,490,000	6,753,968	263,968	4.06730%
2009	1670	6,530,000	6,339,209	-190,790	-2.92175%
2010	2220	7,110,000	6,719,405	-390,594	-5.49360%
2011	3330	7,370,000	7,486,708	116,708	1.58356%

In the next box, we'll see a plot of the linear regression or best-fit line.



The best-fit-line yields the following function:

$$s(p) = 691.264p + 5,184,797.\cdots$$

which seems reasonable. The line and the dots seem to be near each other and the relative errors are all small, with the worst one being 6.20%. Overall, this seems like a relatively good model.

To save time and space, let's assume that Alice has done a similar process to compute worldwide consumption of Silicon, and has the following function:

$$d(p) = -819.472p + 8,238,944.\cdots$$

Since decisions will be made on the basis of Alice's supply function, we should probably check her work. One easy way of checking a linear regression is to see that the (artificial) data point, made by the average x and the average y, actually lies on the best-fit line. This means that if we compute the average price, and put it into $s(p)$, we should get a number very close to the average supply. Another easy way to check a linear regression is to see that the residual errors add up to zero. Those computations are found in the next box.

According to the plan of the previous box, we will now check Alice's regression. The average actual production was

$$\frac{5,650,000 + 6,330,000 + 6,490,000 + 6,530,000 + 7,110,000 + 7,370,000}{6} = 6,580,000$$

while the average price was

$$\frac{1180 + 1440 + 2270 + 1670 + 2220 + 3330}{6} = 2018.3\overline{3}$$

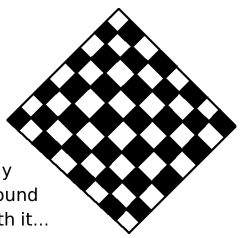
and we can check with

$$s(2018.33) = (691.264)(2018.33) + 5,184,797 = 1,395,198 + 5,184,797 = 6,579,995$$

which is outstandingly close! Meanwhile the sum of the residual errors is

$$350,489 - 149,781 + 263,968 - 190,790 - 390,594 + 116,708 = 0 \text{ (exactly!)}$$

As you can see, Alice's work is excellent.



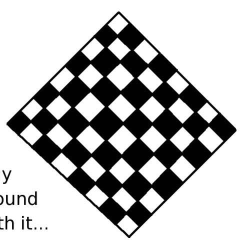
Play Around With it...

3-4-8

Trusting now in Alice's regressions, let's compute the equilibrium price, demand, and supply.

- What is the equilibrium price? [Answer: $ 2021.62.]
- What is the equilibrium demand? [Answer: 6,582,283.$\cdots$.]
- What is the equilibrium supply? [Answer: 6,582,270.$\cdots$.]

Observe that the equilibrium supply and equilibrium demand match to five significant figures, being only off in the sixth. That is the most we can hope for, since we are using six significant figures ourselves in our computations.

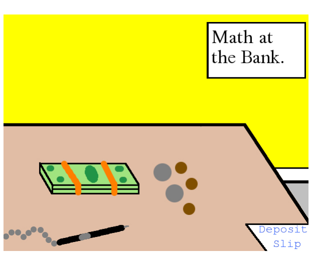


Play Around With it...

3-4-9

Continuing with the previous box, if the price were to be 3000 in the year 2012...

- What would the regressions predict for supply? [Answer: 7,258,589.]
- What would the regressions predict for demand? [Answer: 5,780,528.]
- Is that a shortage or a surplus, and by how much? [Answer: Surplus of 1,478,061.]
- Will the price rise or fall as a result? [Answer: the price will fall.]

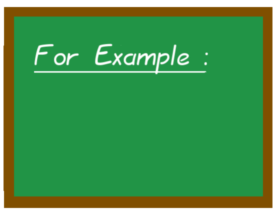


Often, these computations in the real world would be done essentially in the above manner.

1. Raw data would be collected.
2. Regressions would be done to convert those data points into functions.
3. The regressions would be checked, by various means.
4. Predictions would be made using those functions.

It is very tedious to compute linear regressions by hand. In the next few problems, we're going to substitute the process of computing a regression with the process of building a linear model from two data points.

This substitution is neither unrealistic nor artificial. Often a company might have only 2–3 data points available for either supply or demand when attempting to make a casual computation. Generally, data for demand functions can be obtained from a large consumer survey, whereas data for supply functions require a bit more effort to extract (for example, from company websites and annual reports).



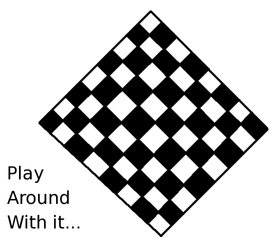
3-4-10

The Wee-Pump Company makes very small pumps suitable for use in aquariums. A new model has been developed, and the marketing department has computed a regression for the demand of the pumps. That demand function is

$$d(p) = 58,000 - 100p$$

but there's not enough data to do a regression for the supply function. The only thing known (by scouring the websites and annual reports of competing companies) is that at the price of \$ 250 the nationwide supply was 46,812 pumps, and at the price of \$ 150 the nationwide supply was 14,512 pumps.

To solve this, we'll first imagine that the supply function is a line connecting the points (150; 14,512) and (250; 46,812). You'll actually do that yourself using the techniques that you learned while reading "Building a Linear Model" on Page 295. Next, you'll find the point where the two functions intersect. You will do this over the next few boxes.

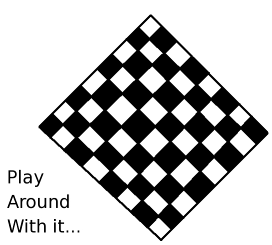


3-4-11

First, let's compute the supply function from the previous example.

- What is the slope of the line connecting the points (150; 14,512) and (250; 46,812)? [Answer: The slope is 323.]
- Using the point (150; 14,512), what is the equation of the supply line, in point-slope form? [Answer: $y - 14,512 = 323(x - 150)$.]
- What is that line in slope-intercept or $y = mx + b$ form? [Answer: $y = 323x - 33,938$.]

We would normally write this supply function as $s(p) = 323p - 33,938$.



3-4-12

Now we have a supply function (from the previous box) and a demand function (from the box before that). Using those, compute the equilibrium.

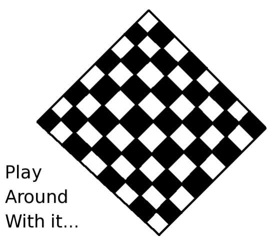
- At equilibrium, what will the price of the new pump be? [Answer: \$ $217.347\cdots$.]
- At equilibrium, what is the demand for the new pump? [Answer: $36,265.2\cdots$ pumps per year.]
- At equilibrium, what is the supply of the new pump? [Answer: $36,265.2\cdots$ pumps per year.]

Just as a reminder, the equilibrium supply and the equilibrium demand should always be the same. The reason I ask for it twice is to encourage you—even force you—to check your work by plugging the equilibrium price into both functions and verifying that they came out the same.

By choosing to build a linear model, we are assuming that the demand function (or supply function) is linear. Is this a safe assumption? How do we know it is linear? In practice, do economists observe behavior that matches the predictions of a linear function?

As it turns out, economists observe a variety of behaviors. Sometimes the demand function behaves like a hyperbola instead of a line, because the demand grows super quickly when the price is low. Sometimes the supply function behaves like a square-root function, instead of a line, because the supply grows sluggishly as the price rises. Parabolas are also used relatively often.

For this reason, it is very important to do some examples with nonlinear functions. We will not be ready to do that, however, until we completely establish our comprehension of this topic at the more basic level of linear functions. Therefore, we will reserve nonlinear functions until the module “Supply and Demand, Part Two, ” beginning on Page 1186.



3-4-13

The Conglomerated Battery Corporation makes batteries for electronic devices. Their research department has carefully modeled the worldwide supply of a particular type of battery (among all suppliers) and obtained the function

$$s(p) = 0.71p - 14.63 \text{ (millions of batteries)}$$

The demand data, however, only extends back two years. Last year the price was \$ 47 each, and 26 million sold. The year before, the price was \$ 61 each and 19 million sold. Using millions of batteries as the unit for quantities and dollars per battery for the cost, compute the following:

- What is the slope of the demand function? [Answer: The slope is -0.5.]
- What is the demand function? [Answer: $d(p) = 49.5 - 0.5p$ (millions of batteries).]
- What is the equilibrium price? [Answer: \$ 53.]
- What is the equilibrium supply? [Answer: 23 million batteries.]
- What is the equilibrium demand? [Answer: 23 million batteries.]

First, we should check that the demand function we produced actually matches the data we are given:

- $d(47) = 49.5 - 0.5(47) = 49.5 - 23.5 = 26$
- $d(61) = 49.5 - 0.5(61) = 49.5 - 30.5 = 19$

Next, we can check that the equilibrium price actually gives the same quantity for both the supply function and the demand function.

- $d(53) = 49.5 - 0.5(53) = 49.5 - 26.5 = 23$
- $s(53) = (0.71)(53) - 14.63 = 37.63 - 14.63 = 23$

Here is a list of all the demand functions that we've seen so far:

- Vacation Rentals: $d(p) = -p + 320$
- Soy: $d(p) = -300p + 400$
- Packages of Pills: $d(p) = -0.1p + 903.8$
- Aquarium Pumps: $d(p) = -100p + 58,000$
- Conglomerated Batteries: $d(p) = -0.5p + 49.5$

Let's take a moment to analyze the demand functions from the previous box. As you can see, all the slopes are negative. This is because the demand always goes up as the price of a product goes down. Sometimes the demand might go up sharply, which is why nonlinear functions like hyperbolas are sometimes used to model demand instead of linear functions. We've not seen that yet in this module, but we'll see it in the second part of Supply & Demand.

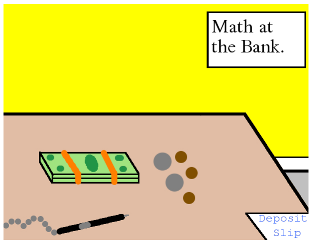
Also, all the y-intercepts for the demand functions are positive. That's because otherwise the demand at price zero (giving it away for free) would be zero or negative, and few products have that behavior (except possibly toxic waste).

Here is a list of all the supply functions that we've seen so far:

- Vacation Rentals: $s(p) = p - 50$
- Soy: $s(p) = 200p - 100$
- Packages of Pills: $s(p) = 15p - 1180$
- Aquarium Pumps: $s(p) = 323p - 33,938$
- Conglomerated Batteries: $s(p) = 0.71p - 14.63$

Now let's take a moment to analyze the supply functions from the previous box. Here again, we can see some interesting features. Firstly, all the slopes are positive; this is because the supply always goes up as prices go up, though it may go up very slowly. It is for this reason that sometimes nonlinear supply functions are used, such as a square root. We haven't seen that yet, but we will see it in the second Supply & Demand module.

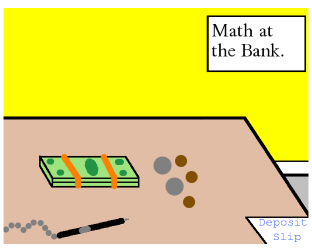
Additionally, all the y-intercepts are negative. That's because if the y-intercept were positive, then $s(0)$ would be positive. How could that be? Why would a company go through the trouble of manufacturing a product and all the incumbent costs in time and money, only to give it away for free? Of course, it is vaguely possible that a company might do this to introduce their product to a new market, but in this case they are being compensated in the form of free advertising, which does have a dollar value. In any case, I'm sure we can agree that $s(0)$ should not be positive, and so the y-intercept cannot be positive.



Most of my readers shop relatively often, and so they are familiar with how the price of a good affects their own desire for it—from this it is easy to extend to the idea of market demand fluctuating with price.

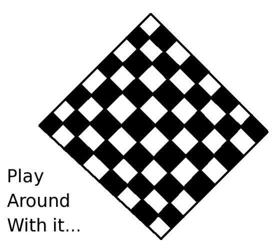
The fluctuation of supply with price is harder to grasp, as most students have not been manufacturing anything. Therefore, imagine yourself in the role of an intern in the production department of a company making a sports drink. If the going market price for sports drinks begins to increase, you might consider going from two shifts a day to three shifts a day (i.e. 24 hours a day) at your factory. While there would be overtime costs, it might be very profitable if the price goes high enough. You could also consider diverting resources from other projects, such as children's beverages or dessert drinks, if those are less profitable. Both of these moves would increase the quantity of sports beverage that your company is producing, and thus increase the supply.

If the price gets high enough, then companies that manufacture related products, such as ketchup or coffee, might look into the feasibility of opening up a sports-drink side business. This would also definitely increase the supply.



Probably one of the most dramatic examples of supply increasing with price has to do with the price of gold. Over the last twenty years, the price of gold has increased dramatically—though not steadily—and this has caused several mining companies to reexamine very old and long-closed gold mines in California and Nevada. While the cost of reopening a mine in 2012 that has been closed since 1956 would be enormous, if the price of gold goes high enough it would be very profitable to do so.

You can read more about this in the article "In New California Gold Rush, Old Mines Reopen" by Jesse McKinley, published in *The New York Times* on February 10th, 2011.



3-4-14

Silver nitrate is used in many industrial areas, including photography, controlled explosives, and electron microscopes. Let us suppose that in 2011 there were 10,773,000 kg of silver nitrate used worldwide, but in 2012 there were 11,738,800 kg used. The price of silver nitrate was \$ 1430 per kg in 2011, but fell to \$ 1385 per kg in 2012. Finally, the amount of silver nitrate manufactured worldwide was 12,542,900 kg in 2011, and 11,708,600 kg in 2012. (For simplicity of computation, let's measure the quantities in thousands of kg, and the price in dollars per kg.) Over the course of the next three boxes, compute answers to the following questions:

- What was the change in price from 2011 to 2012? [Answer: -\$ 45 per kg.]
- What was the change in the demand from 2011 to 2012? [Answer: +965,800 kg, or -965.800 (thousands of kg).]
- What is the slope of the demand function? [Answer: $-21.4622\cdots$.]
- What is the equation of the demand line, in point-slope form? [Answer: either $(d-10,773.0) = -21.4622(p-1430)$ or $(d-11,738.8) = -21.4622(p-1385)$.]
- What is the equation of the demand line, in slope-intercept form? [Answer: $d(p) = -21.4622p + 41,463.9$.]

Play
Around
With it...

3-4-15

Continuing with the previous box...

- What is the change in supply from 2011 to 2012?
  [Answer: -834,300 kg, or -834.300 (thousands of kg).]
- What is the slope of the supply function? [Answer: 18.54.]
- What is the equation of the supply line, in point-slope form?
  [Answer: either $(s - 12,542.9) = 18.54(p - 1430)$
  or $(s - 11,708.6) = 18.54(p - 1385)$.]
- What is the equation of the supply line, in slope-intercept form?
  [Answer: $s(p) = 18.54p - 13,969.3$.]

Play
Around
With it...

3-4-16

Still continuing with the previous box...

- What is the equilibrium price of silver nitrate? [Answer: \$ $1385.75\cdots$.]
- What is the equilibrium demand of silver nitrate? [Answer: $11,722.6\cdots$.]
- What is the equilibrium supply of silver nitrate? [Answer: $11,722.5\cdots$.]

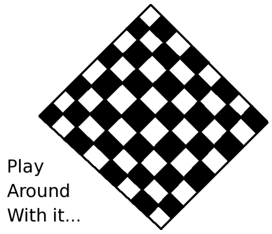
  (Note, the difference of 0.1 between equilibrium demand and equilibrium supply is due to rounding error.)
- Was the market in surplus or shortage in 2011, and by how much?
  [Answer: Surplus of 1,769,900 kg.]
- Was the market in surplus or shortage in 2012, and by how much?
  [Answer: Shortage of 30,200 kg.]

Play
Around
With it...

3-4-17

In a medium-sized town a physical therapist has been running a massage service as a side business. The standard way of doing this is to split the fees with the masseurs and masseuses that he hires. If the group charges \$ 60 per massage, then they have 18 customers per week. However, if the group charges \$ 40 per massage, then they have 58 customers per week. However, only 3 masseurs/masseuses are willing to work at the \$ 40 price, while 5 masseurs/masseuses are willing to work at the \$ 60 price. (This should make sense, because in the first case they each pocket \$ 20, and in the other they each pocket \$ 30.) Assume that each masseur/masseuse can perform 6 massages per week. Now over the next two boxes, compute the following supply and demand data:

- What is the demand (in massages per week) at \$ 40 per massage?
  [Answer: 58 massages.]
- What is the demand (in massages per week) at \$ 60 per massage?
  [Answer: 18 massages.]
- What is the slope of the demand function? [Answer: The slope is -2.]
- In $d(p) = mp + b$ form, what is the demand function? [Answer: $d(p) = -2p + 138$.]



3-4-18

Continuing with the previous box...

- What is the supply (in massages per week) at \$ 40 per massage? [Answer: 18 massages.]
- What is the supply (in massages per week) at \$ 60 per massage? [Answer: 30 massages.]
- What is the slope of the supply function? [Answer: The slope is 0.6.]
- In $s(p) = mp + b$ form, what is the supply function? [Answer: $s(p) = 0.6p - 6$.]
- What is the market equilibrium price? [Answer: \$ $55.3846\cdots \approx$ \$ 55.38.]

We can check that result by looking at $d(p)$ and $s(p)$ at $p = 55.3846$.

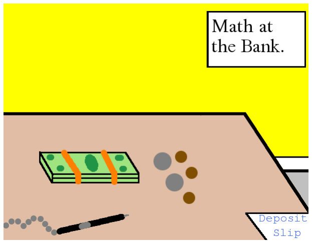
- $d(55.3846) = 138 - 2(55.3846) = 138 - 110.769\cdots = 27.2308\cdots$
- $s(55.3846) = -6 + 0.6(55.3846) = -6 + 33.2307\cdots = 27.2307\cdots$

Other than rounding error, we obtain an exact match, and so we can be confident that we found the equilibrium correctly, assuming we built the correct functions. Note, we use the fictional price of \$ 55.3846, even though one can charge \$ 55.38 or \$ 55.39, but not \$ 55.3846.

Incidentally, I made an error when typing the previous checkerboard and check-your-work boxes. It was only by checking my work that I found the error—otherwise the book might have been published with a mathematical error, which could have been very embarrassing for me. This highlights the need to always check your work.

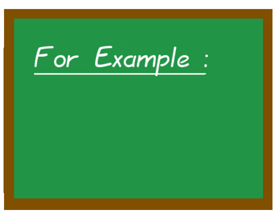
Our check of the equilibrium point was contingent on our building the correct supply and demand functions. How can we check those? We will plug in the data points that we were given at the start of the problem.

- $d(40) = 138 - 2(40) = 138 - 80 = 58$ (an exact match)
- $d(60) = 138 - 2(60) = 138 - 120 = 18$ (an exact match)
- $s(40) = -6 + 0.6(40) = -6 + 24 = 18$ (an exact match)
- $s(60) = -6 + 0.6(60) = -6 + 36 = 30$ (an exact match)



We might worry about the impact of rounding 55.3846 to \$ 55.38 or \$ 55.39. Normally a business would probably charge either \$ 55 or \$ 56 because it sounds "rounder," but will that cause the economic equilibrium to stop working? Let's investigate.

- $d(55) = 138 - 2(55) = 138 - 110 = 28$ massages per week.
- $s(55) = -6 + 0.6(55) = -6 + 33 = 27$ massages per week.
- At \$ 55 we see a shortage of 1 massage per week. (This means that there is one person who wants a massage but cannot get an appointment.)
- $d(56) = 138 - 2(56) = 138 - 112 = 26$ massages per week.
- $s(56) = -6 + 0.6(56) = -6 + 33.6 = 27.6$ massages per week.
- At \$ 56 we see a surplus of 1.6 massages per week. (This means that there are 1.6 open appointment slots which never get taken.)
- In either case, the situation is really close to an equilibrium.



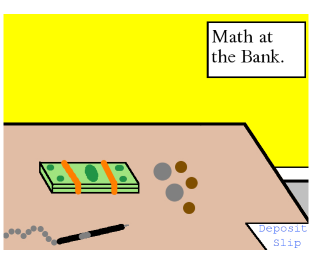
3-4-19

An important question comes up frequently in Supply & Demand analysis: what is the highest price that can be charged? What this means is "what value of p will cause $d(p)$ to be some positive number, no matter how small?" Let's take the soy problem from Page 438 as our opening example.

To compute this, we solve $d(p) > 0$ for p. For the soy problem, we had $d(p) = -300p + 400$ so we can proceed as follows:

$$\begin{aligned} d(p) &> 0 \\ -300p + 400 &> 0 \\ -300p &> -400 \\ p &< \frac{-400}{-300} \\ p &< 1.33\overline{3} \end{aligned}$$

Therefore, we learn that the demand will be positive so long as $p < 1.33\overline{3}$. We can call this number (\$ $1.333\cdots$) the *maximum feasible price* because it is the highest price that anyone is willing to pay. (No one is willing to pay more than that.)



To put the information from the previous box into a more economic light, if the price of soy is less than $1.33\overline{3}$, then we expect the demand for soy to be some positive number. If it is exactly $1.33\overline{3}$, then the demand would be exactly zero. Of course, a price with a non-terminting decimal is impractical and unrealistic, as commercial prices usually are to the nearest penny, and industrial prices are usually to the nearest tenth of a cent.

The troublesome part would be if $p > 1.33\overline{3}$. The mathematical function $d(p)$ has values for any number p. However, for $p > 1.33\overline{3}$ we know that $d(p)$ will be a negative number. In most situations a negative demand makes no sense. Instead, negative demand is intended to be thought of as zero demand. Economists and mathematicians approach this apparent contradiction differently, as we will see in the next two boxes.

Most mathematicians would say that the above demand function really should look like the following:

$$d(p) = \begin{cases} 1.33\overline{3} \leq p < \infty & 0 \\ 0 \leq p \leq 1.33\overline{3} & -300p + 400 \end{cases}$$

The majority of readers will have never seen this notation before, and therefore this requires some explanation. The big { symbol means that $d(p)$ will behave differently depending if p satisfies the inequality $1.33\overline{3} \leq p < \infty$ or if p satisfies the inequality $0 \leq p \leq 1.33\overline{3}$. You can see that all p values that are not negative will fall into one category or the other. For prices between $1.33\overline{3}$ and infinity, the demand is zero. For prices between 0 and $1.33\overline{3}$, the price is given by computing $-300p + 400$.

Written this way, the function has a domain of all p such that $0 \leq p < \infty$, or all non-negative numbers p. Last but not least, you can see that $1.33\overline{3}$ falls into both categories. That's not a problem, because

$$-300\left(1.33\overline{3}\right) + 400 = 0$$

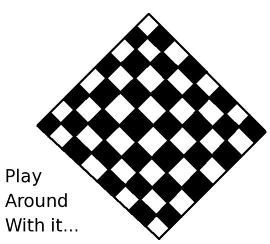
so the two formulas given actually agree on what $d(1.33\overline{3})$ should be. This function has nothing to say about negative values of p.

I'm sure this box must be very confusing for many readers. That's okay, because we won't be using this notation in this textbook, except in the module "Piecewise Functions and Progressive Taxation," starting on Page 1244. Instead we will use the notation that most economists appear to use while teaching, which is explained in the next box.

Some economists will approach things as mathematicians do, using the notation of the previous box. However, the majority of economists appear to just tag the function with an explicit definition of the applicable domain of p. They would write the demand function as

$$d(p) = -300p + 400, \text{ for all } 0 \leq p \leq 1.33\overline{3}$$

which is truly more compact and much easier to include in an email or a presentation. The "$0 \leq$" part means that prices which are negative are not allowed; likewise, the "$\leq 1.33\overline{3}$" part means that prices which are above $1.33\overline{3}$ are also not allowed. It is assumed that the reader will understand that the demand shall become zero when the price grows very high.

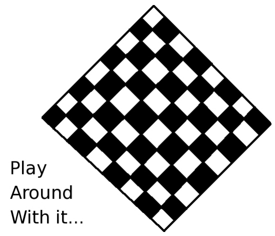


3-4-20

Now let's return to the example of the demand function for the aquarium pumps, as described on Page 443. The demand function was $d(p) = 58,000 - 100p$.

- What inequality must we solve to determine when the demand is positive? [Answer: We must solve $58,000 - 100p > 0$.]
- What do we get when we solve that inequality for p? [Answer: $p < 580$.]
- What is the maximum feasible price for these aquarium pumps? [Answer: \$ 580.00.]

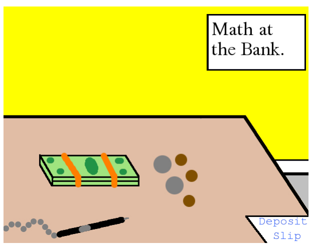
If you ever need a shortcut for computing the maximum feasible price, just solve $d(p) = 0$. For reasons that will be explained to you in a few boxes, you should always round downward to the next penny.



3-4-21

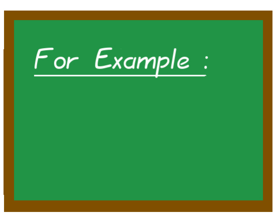
Using the table of demand functions on Page 445 as a reference, find the following maximum feasible prices:

- What is the maximum feasible price for the vacation rentals? [Answer: \$ 320 per week.]
- What is the maximum feasible price for the allergy pills? [Answer: \$ 9038.00 per package.]
- What is the maximum feasible price for the batteries? [Answer: \$ 99.00 per battery.]



Once the price of a manufactured item goes below a certain threshold, it is no longer profitable to manufacture it. An example of this would be obsolete audio technology. Certainly there are enthusiasts willing to collect 8-track tapes, cassette tapes, and vinyl records. The same can be said about VHS video tapes. However, most people have abandoned these obsolete technologies. What does this do to the market for the devices which play these media? There is some demand for these devices from collectors, but enough of the devices are available used—with very low demand—so the price cannot go very high at all. Therefore it makes no sense to devote a factory to making new players for these old technologies.

The interesting question is if we can compute the price at which manufacturing no longer makes sense. This will be the *minimum feasible price*.



3-4-22

To explore the topic of the minimum feasible price, let's consider the aquarium pumps. We had a supply function of $s(p) = 323p - 33,938$. We're going to determine what values of p have $s(p) > 0$. In other words, for what prices will there be anyone at all willing to make pumps of this variety.

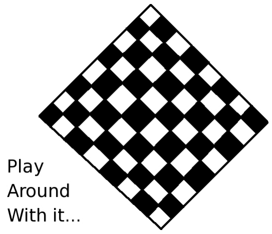
$$\begin{aligned} s(p) &> 0 \\ 323p - 33,938 &> 0 \\ 323p &> 33,938 \\ p &> 105.071\cdots \end{aligned}$$

Thus we learn that so long as $p > 105.071$ we will have $s(p) > 0$. In other words, $p > 105.071$ will result in some supply, no matter how small. For this reason, we will say that \$ 105.08 is the *minimum feasible price* for these aquarium pumps.

We can easily check our work from the previous box.

- $s(105.07) = 323(105.07) - 33,938 = 33,937.61 - 33,938 = -0.39$
- $s(105.08) = 323(105.08) - 33,938 = 33,940.84 - 33,938 = 2.84$

As you can see, \$ 105.07 results in a negative supply, which is probably not meaningful. However, \$ 105.08 results in a positive supply. Therefore, the minimum feasible price is \$ 105.08. As you can see, we must always round the minimum feasible price *upward* to the next penny, and the maximum feasible price *downward* to the next penny. You must not round to the *nearest* penny, as that would have produced a wrong answer here.

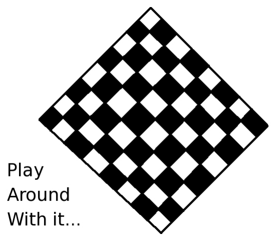


\# 3-4-23

Now let's return to the example of the supply function for the allergy pills, as described on Page 438. The supply function was $s(p) = 15p - 1180$.

- What inequality must we solve to determine when the supply is positive? [Answer: We must solve $15p - 1180 > 0$.]
- What do we get when we solve that inequality for p? [Answer: $p > 78.6\overline{6}$.]
- What is the minimum feasible price for these aquarium pumps? [Answer: \$ 78.67.]

If you ever need a shortcut for computing the minimum feasible price, just solve $s(p) = 0$, and round upward to the next penny.



\# 3-4-24

Using the table of supply functions on Page 445 as a reference, find the following minimum feasible prices:

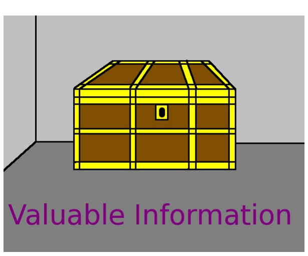
- What is the minimum feasible price for that variety of soy? [Answer: \$ 0.50 per pound.]
- What is the minimum feasible price for the vacation rentals? [Answer: \$ 50 per week.]
- What is the minimum feasible price for the batteries? [Answer: \$ 20.61 per battery.]

Using the Conglomerated Batteries example, to properly write their supply function in the economist notation, we would write

$$s(p) = 0.71p - 14.63 \text{ for all } p \geq 20.61$$

whereas a mathematician would write

$$s(p) = \begin{cases} 20.61 \leq p < \infty & 0.71p - 14.63 \\ 0 \leq p \leq 20.61 & 0 \end{cases}$$

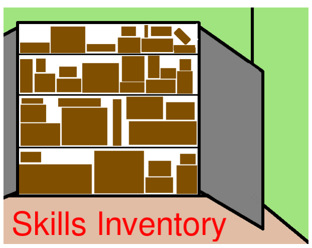


Students who like to think graphically might want to flip back to Page 439 and look at the three plots of the Supply & Demand functions there. Look at the x-intercepts—the coordinates of the x-intercepts are the minimum and maximum feasible prices! The minimum feasible price is where the supply curve intersects the x-axis. The maximum feasible price is where the demand curve intersects the x-axis.

The x-axis, restricted to positive values, represents all possible prices. However, the only prices that will result in any exchange—purchasing or manufacturing—would be those between the minimum and maximum feasible price. Below the minimum feasible price, there is no supply; above the maximum feasible price, there is no demand.

Here I will write a short biography of Adam Smith (1723–1790), who developed the theory of supply and demand, and the notion of "the invisible hand."

However, I ran out of time and haven't written it yet.



The first part of our exploration of Supply & Demand has now ended. We have gained the following skills/knowledge in this module:

- How to calculate supply or demand at a given price.
- How to determine the equilibrium price by equating the supply and demand functions.
- The economic effects of a shortage or a surplus on an industry, especially with regrard to consumer and producer behavior.
- How to develop a linear model of supply (or demand) when two data points are known.
- How to develop a linear model of supply (or demand) after performing a regression on collected data.
- The differing characteristics of supply and demand lines (e.g. slopes and intercepts.)
- An introduction to the idea of some goods having inelastic demand (e.g. life-saving medicines) while others have elastic demand.
- How to calculate the minimum feasible price for an industry when the supply function is known.
- How to calculate the maximum feasible price for an industry when the demand function is known.
- As well as the vocabulary terms: elastic demand, elasticity of demand, equilibrium, equilibrium demand, equilibrium price, equilibrium supply, inelastic demand, maximum feasible price, minimum feasible price, shortage, surplus.