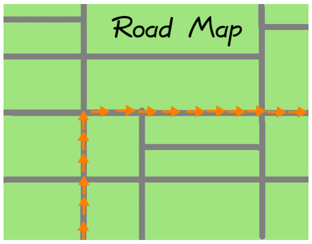
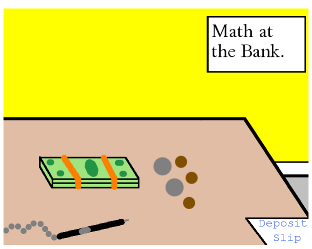
Module 3.6: Inflation and the True Rate of Return



Here you will learn about inflation, one of the famous forces of economics, as well as how to adjust interest rates for its effects. We will also talk about how to analyze historical data. Many studies of trends can be highly misleading when not adjusted for inflation. We will consider the minimum wage as our example. Finally, we will learn about Fisher's Equation, its uses, its derivation, and how it allows us to compute "the real rate of return."

Prices in previous decades were very different from where they are today. The process by which prices adjust is called inflation. Some items change in price differently than others, of course. For example, radios were once very expensive indeed, and a family would probably have one radio or none, but not two. Radios today are very cheap, however. Thus, we can safely say that the prices of radios have fallen. (This is primarily because technology has made them easier to manufacture, but also because they have become smaller and lighter, which is important for shipping.)

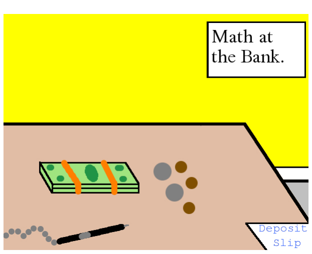
Such *decreases* are unusual—almost all objects *increase* in price over time. Furthermore, most objects will increase in price over time according to a measurable and slow rate.



Inflation is about measuring how the prices of things rise over time. Similarly, that means inflation is about how the purchasing power of a fixed dollar amount decreases over time. The purchasing power of \$ 100 is much smaller now than it was 50 years ago. Sometimes you can detect this in old TV shows. I used to watch *M*A*S*H* often, a TV show describing the Korean War (1951–1953), and I noticed that there was a roughly 1:8 ratio in prices from then to when I was watching it in the early 1990s. Luckily, we don't have to guess or make estimates when it comes to measuring the progress of inflation over the years. The US Government has been carefully tracking inflation since 1913.

When making forecasts into the future, or when analyzing past data, it is vital to correct for inflation. Otherwise, the data can seem meaningless. For example, when planning your retirement, you are making calculations about what will happen when you are 65—prices then will be very different from prices now.

Alternatively, if you want to watch how the minimum wage has altered over time, then you have to realize the impact of inflation on prices. In 1933, when the minimum wage was first created, the minimum wage was 25 cents per hour. What does earning 25 cents per hour in 1933 mean in terms of 2010 dollars? Inflation will tell us.



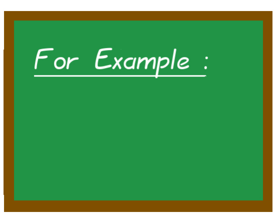
To measure past inflation, the US government has made available the *consumer price index* or CPI. The CPI is a very carefully crafted index, monitoring the price of all sorts of consumer objects. A direct proportion is all that is needed to change from the price in one year to the price in another.

As we will discuss later, the CPI is not just a collection of consumer products in the sense of what can be bought at a department store. It includes medical expenses, food, gasoline, and even education. Incidentally, regional data is also available, as inflation affects geographically-separated cities in the USA at slightly different rates.

The Bureau of Labor Statistics, an agency of the US Federal Government, has made this historical data available. Below is the Consumer Price Index for each year since 1913.

Year	CPI	Year	CPI	Year	CPI	Year	CPI	Year	CPI	Year	CPI
1913	9.8	1930	17.1	1947	21.5	1964	30.9	1981	87.0	1998	161.6
1914	10.0	1931	15.9	1948	23.7	1965	31.2	1982	94.3	1999	164.3
1915	10.1	1932	14.3	1949	24.0	1966	31.8	1983	97.8	2000	168.8
1916	10.4	1933	12.9	1950	23.5	1967	32.9	1984	101.9	2001	175.1
1917	11.7	1934	13.2	1951	25.4	1968	34.1	1985	105.5	2002	177.1
1918	14.0	1935	13.6	1952	26.5	1969	35.6	1986	109.6	2003	181.7
1919	16.5	1936	13.8	1953	26.6	1970	37.8	1987	111.2	2004	185.2
1920	19.3	1937	14.1	1954	26.9	1971	39.8	1988	115.7	2005	190.7
1921	19.0	1938	14.2	1955	26.7	1972	41.1	1989	121.1	2006	198.3
1922	16.9	1939	14.0	1956	26.8	1973	42.6	1990	127.4	2007	202.4
1923	16.8	1940	13.9	1957	27.6	1974	46.6	1991	134.6	2008	211.1
1924	17.3	1941	14.1	1958	28.6	1975	52.1	1992	138.1	2009	211.1
1925	17.3	1942	15.7	1959	29.0	1976	55.6	1993	142.6	2010	216.7
1926	17.9	1943	16.9	1960	29.3	1977	58.5	1994	146.2	2011	220.2
1927	17.5	1944	17.4	1961	29.8	1978	62.5	1995	150.3	2012	226.7
1928	17.3	1945	17.8	1962	30.0	1979	68.3	1996	154.4	2013	230.3
1929	17.1	1946	18.2	1963	30.4	1980	77.8	1997	159.1	2014	233.9

You can find this data at: `http://www.bls.gov/cpi/`



\# 3-6-1

In 1962 a friend of mine's father was selling his bachelor's pad in New York City. He had invested \$ 500 in replacing all the old kitchen appliances with new ones, and got a great price for his place, which he attributes partially to the redoing of the kitchen. He suggested that I do the same when I left Maryland for New York in 2007, but what would the equivalent investment have been?

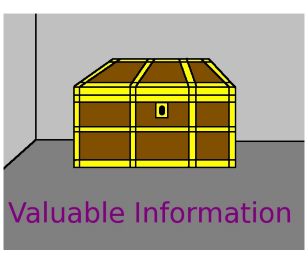
The CPI in 1962, based on the table, was 30.0 and in 2007 it was 202.4. Now to convert the prices, all that is required is a direct proportion. We would write

$$\frac{\text{New cost} \rightarrow \qquad x}{\text{Old cost} \rightarrow \qquad 500} = \frac{202.4 \qquad \leftarrow \text{New CPI}}{30.0 \qquad \leftarrow \text{Old CPI}}$$

and solve for the equivalent sum, which would be

$$(500.00)\frac{202.4}{30.0} = 500 \times 6.746\overline{6} = 3373.33$$

We can conclude that the equivalent investment would be about \$ 3733.33. Of course, sometimes wealthy people spend much more than that on renovating a kitchen.



To convert prices between two years in the past or present, using the CPI to model inflation, use the formula

$$\frac{\text{CPI in Year A}}{\text{CPI in Year B}} = \frac{\text{Price in Year A}}{\text{Price in Year B}}$$

which is equivalent to

$$(\text{Price in Year B})\,\frac{\text{CPI in Year A}}{\text{CPI in Year B}} = \text{Price in Year A}$$

Play Around With it...

\# 3-6-2

When I got my first job after graduating RPI in 1999, my salary was \$ 45,500 per year. What is that salary in 2010 dollars? [Answer: \$ 60,011.25 per year.]

Note: I earned a degree in engineering. You should not expect the same starting salary when you graduate college, if you are majoring in a different subject, such as philosophy.

Play Around With it...

\# 3-6-3

My boss's boss's boss had a salary then (in 1999) of around \$ 110,000 per year.

- How much is that worth in 2010? [Answer: $\$ 145,082$.]
- How much would the equivalent have been in 1933, per week? [Answer: \$ 166.08 per week.]

Play Around With it...

\# 3-6-4

In a science-fiction story I once wrote, a young man (in 2010) learns of his great-great-uncle who was killed at age 19 fighting for the USA in the First World War. He travels back in time to warn his relative not to enlist. He has \$ 200 in his wallet when he leaves.

Ignoring the fact that monetary bills with a year 2010 date would not be legal tender in 1915, how much money would this \$ 200 be worth in 1915? [Answer: \$ 4291.08.]

For Example :

\# 3-6-5

A regular church-goer, born in 1940, has always placed \$ 2 in the collection plate since he was 18 years old, each week. However, he never adjusted his donation to take into account inflation. What value should he have put in the collection plate in 1968, 1978, 1988, 1998, and 2008 to match the true purchasing power of his \$ 2 from 1958? In each case, what percentage of that amount would his actual \$ 2 donation represent?

In 1958:	$(2.00)\,(28.6)\,/\,(28.6)$	$=$	2.00	$100 \times (2.00)\,/\,(2.00)$	$=$	100.00%
In 1968:	$(2.00)\,(34.1)\,/\,(28.6)$	$=$	2.38	$100 \times (2.00)\,/\,(2.38)$	$=$	$84.03\cdots\%$
In 1978:	$(2.00)\,(62.5)\,/\,(28.6)$	$=$	4.37	$100 \times (2.00)\,/\,(4.37)$	$=$	$45.76\cdots\%$
In 1988:	$(2.00)\,(115.7)\,/\,(28.6)$	$=$	8.09	$100 \times (2.00)\,/\,(8.09)$	$=$	$24.72\cdots\%$
In 1998:	$(2.00)\,(161.6)\,/\,(28.6)$	$=$	11.30	$100 \times (2.00)\,/\,(11.30)$	$=$	$17.69\cdots\%$
In 2008:	$(2.00)\,(211.1)\,/\,(28.6)$	$=$	14.76	$100 \times (2.00)\,/\,(14.76)$	$=$	$13.55\cdots\%$

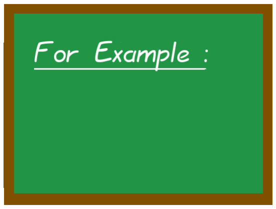
As you can see, this means he's giving around 1/7th now (in the inflation-adjusted sense), of what he used to give when he was a teenager.

Play Around With it...

\# 3-6-6

The phrase "become a millionaire" appears on the box of a very popular board game from The Great Depression. (You know which game I mean, but I cannot type its name into the book without paying a royalty fee to that game's publishing company.)

What dollar amount in 2010 is equivalent to a million dollars of 1933? [Answer: \$ 16,798,449.]

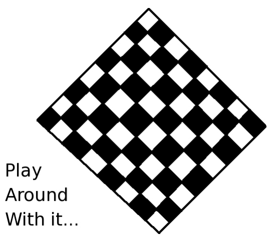


3-6-7

There's a little bit of false precision in the previous box, because we cannot possibly know the equivalent purchasing price to ten significant figures. The way that this is handled in real life is to use an interval, which we'll learn much more about in the inequalities section. When the CPI for 1933 is listed as 12.9, we know that means it is between 12.85 and 12.95. When the CPI for 2010 is listed as 216.7, we know that means it is between 216.65 and 216.75. Therefore, the purchasing power of one million 1933 dollars in this present year would be between

$$\begin{aligned} 1{,}000{,}000.00 \times (216.65/12.95) &= 1{,}000{,}000.00 \times 16.7297\cdots = 16{,}729{,}729 \\ 1{,}000{,}000.00 \times (216.75/12.85) &= 1{,}000{,}000.00 \times 16.8677\cdots = 16{,}867{,}704 \end{aligned}$$

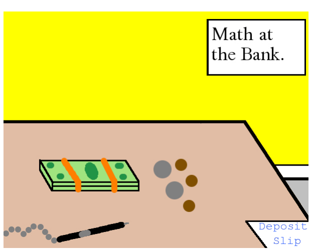
and so the most justifiable answer would be "roughly between 16.7 and 16.9 million dollars." Notice the two fractions are (lowest/highest) and (highest/lowest), and that will always be the case.



3-6-8

Repeat the above problem with 1913 instead of 1933 and with 2014 instead of 2010. That is to say, what is the interval representing the purchasing power of one million 1913 dollars in the money of 2014? Round to the nearest thousand dollars.

[Answer: $ 23,741,000 and $ 23,995,000.]



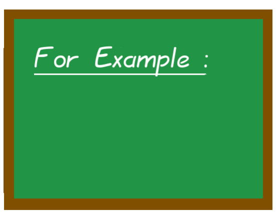
Perhaps you might be wondering how the CPI is computed? The following quote is found on the website of the Bureau of Labor Statistics, and describes what is in the "all urban consumers" version of the consumer price index (called CPI-U) that was given in the large table several pages ago.

- Food and beverages (ham, eggs, carbonated drinks, coffee, meals and snacks);
- Housing (rent of primary residence, fuel oil, bedroom furniture);
- Apparel (men's shirts and sweaters, women's dresses, jewelry);
- Transportation (new vehicles, gasoline, tires, airline fares);
- Medical care (prescription drugs and medical supplies, physicians' services, eyeglasses and eye care, hospital services);
- Recreation (television sets, cable TV, pets and pet products, sports equipment, admissions);
- Education and communication (college tuition, postage, telephone services, computer software and accessories);
- Other goods and services (tobacco and smoking products, haircuts and other personal care services, funeral expenses).

You can find that at: `http://www.bls.gov/cpi/cpifact8.htm`

Of course, when you're trying to predict for the future, you cannot know what inflation will be like. It is driven by so many factors, such as the rate of unemployment, the size of the budget deficit, and currency valuation—but the three most commonly quoted values for forecasting inflation are 3%, 3.5%, and 4%. Later, we will see if these values are wise, or not.

First, let's examine why we cannot use the CPI to forecast into the future. That's because the CPI is calculated based on a survey of actual prices in actual stores. Therefore, the CPI ten years from now cannot be known in advance. Second, we must also remember that if we forecast 20 or more years out, that the forecast cannot be accurate to the penny. Instead, long-term forecasts must always be taken with a grain of salt.



3-6-9

Suppose that I want to buy a boat. The boat costs \$ 25,000 now, and I need to save up for five years. Thus, I should then aim for the value that the boat will cost *then*, and not the value the boat will cost *now*. How can I compute what the boat will cost in five years? To answer this question, we could use the 3% and 4% models, and see what happens. Therefore, we're going to see how much the boat would cost under the 3% model, and under the 4% model.

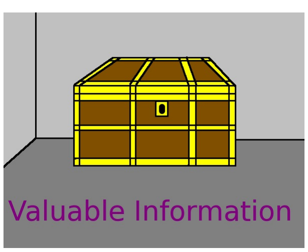
Remember, three precent inflation means that we are raising the price by 3%. Therefore, with three percent, after one year the boat should cost $25,000 \times 1.03 = 25,750$; and after 2 years $25,750 \times 1.03 = 26,522.50$. Likewise for 3 years $26,522.50 \times 1.03 = 27,318.18$, for 4 years $27,318.18 \times 1.03 = 28,137.72$, and finally for 5 years $28,137.72 \times 1.03 = 28,981.85$. Of course, now that we know the laws of exponents, we can write

$$25,000 \times (1.03)^5 = (25,000)(1.15927\cdots) = 28,981.8\cdots$$

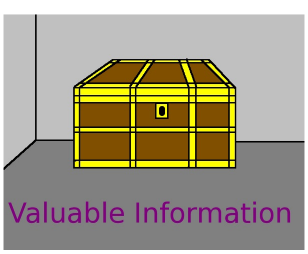
and then for four percent

$$25,000 \times (1.04)^5 = (25,000)(1.21665\cdots) = 30,416.3\cdots$$

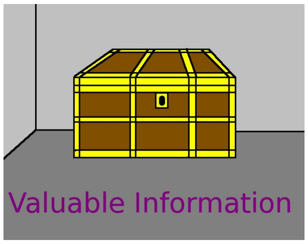
Thus, I can be relatively confident that the boat will cost between \$ 28,981 and \$ 30,416 five years from now, if I am confident that inflation will be between 3% and 4% over those periods of time.



- From the previous example, we see that when looking t years into the future the cost of an object now costing D dollars is predicted to become $D\,(1 + r_{infl})^t$ dollars.
- Some students are quick to notice that this is just compound interest—the interest rate r is just the rate of inflation r_{infl}. The cost of the object today becomes P and the future cost becomes A. The compounding is annual, and t takes its usual meaning—the number of years.

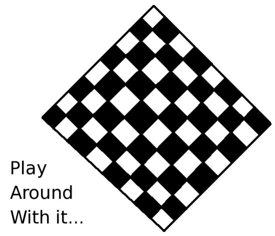


- Similarly, the purchasing power of D dollars, t years in the future, can be predicted to be equal to $D\,(1 + r_{infl})^{-t}$ in today's dollars.
- Again, some students are quick to notice that this is also compound interest—the interest rate r is just the rate of inflation r_{infl}. The purchasing power today of the money is P and the amount of money in the future is A. Once again, the compounding is annual.



On the other hand, when working with the past, one can simply look up the actual CPI values and calculate the effects of inflation precisely!

Inflation is unusual in that it has this split personality—depending on if the problem involves looking backward into the past or forward into the future.

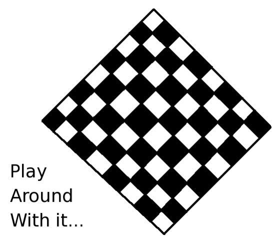


3-6-10

Fred's grandparents have given him a cash gift of \$ 3500 on the day he graduates college at the age of 22. He has lots of expenses relating to moving to his new job and buying professional clothing, so he's very grateful.

About 60 years later, suppose that Fred's grandchild is now graduating college. Fred remembers how useful that gift of cash was when he graduated, and he wants to do the same. How much would you forecast that he should give, to have roughly an equal impact in terms of purchasing power? Use the 3% and 4% estimates for your prediction.

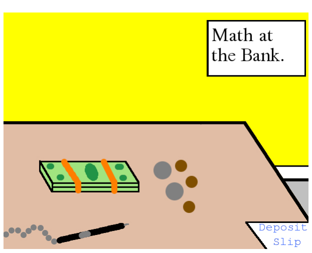
[Answer: between \$ 20,620 and \$ 36,818.]



3-6-11

Suppose that a widow, aged 65, has been left \$ 2,000,000. Instead of putting it in a bank, which she distrusts, she hides it in various places around the house. Further suppose that when she passes away in the future, at the age of 100, the cash is found, and it is discovered that she has spent none of it. Approximately what will the purchasing power be, in today's dollars, of those two million dollars? To be open to all eventualities, assume inflation is between 2.5% and 4.5% per year.

[Answer: between \$ 842,742. $\cdots$, and \$ 428,508. $\cdots$, depending on inflation.]



The previous box explains the principal reason that I want you to know about the principle of inflation. Over a medium-length timespan of 35 years, the two million dollars "decayed" into effectively between 842k and 428k. Your money is slowly but continuously losing purchasing power. This is why investment is, for all intents and purposes, mandatory. If you do not invest, you cannot resist the forces of inflation. In fact, inflation is working against you.

Furthermore, the above is to say nothing of the risk of fire, theft, or the slow destruction of wealth due to occasional expenditure. As you can see, the value of money depreciates quite remarkably, just from the passage of time.

There are two important mistakes that many people make in regards to inflation. The first is to make financial planning decisions without taking inflation into account. This is obviously very unwise and misleading. The second is to realize that when you have inflation by 4%, while prices rise by 4%, it is not the case that purchasing power decreases by 4%. We will explore that second point with a quick example.

For Example :

\# 3-6-12

Consider someone with a suitcase containing \$ 100,000. He decides to keep it in his bedroom for one year, so that he can admire it. Assuming that inflation is 4%, what will the purchasing power of that \$ 100,000 be, one year from now?

We can quote the formula given earlier:

$$D(1+r_{infl})^{-t} = (100,000)(1+0.04)^{-1} = 96,153.8\cdots$$

or we can solve a compound interest problem:

$$\begin{aligned} A &= P(1+i)^n \\ 100,000 &= P(1+0.04)^1 \\ 100,000 &= P(1.04) \\ \frac{100,000}{1.04} &= P \\ 96,153.8\cdots &= P \end{aligned}$$

What we cannot do is simply say that the purchasing power is 4% less. That would imply \$ 96,000 on the dot, which is incorrect. The discrepancy is small, but this is a favorite trap that can be commonly found on exams, so beware.

For Example :

\# 3-6-13

Over extremely long intervals of time, the stock market has roughly grown at 9% per year. Suppose Rich invested some money that he received as a graduation present (perhaps \$ 2000) in an IRA. If he saves it until he is 66, and he is 22 when he receives his Bachelor's degree, then how much would Rich have at age 66? If you consider 4% inflation, how much is that worth, in today's dollars?

First, let's find the amount at the end. We would have

$$A = P(1+i)^n = (2000)(1+0.09)^{66-22} = (2000)(1.09)^{44} = 88,673.91$$

which is very impressive. He was given \$ 2000—a sum of money insufficient to purchase any but the most used of cars—and it turns into enough for a very nice expensive car, or so we might imagine.

However, we must adjust for inflation! We will do that in the next box.

Continuing with the previous box, we would have $r_{infl} = 0.04$ and $t = 44$. Let's try the compound interest method for predicting inflation.

$$\begin{aligned} A &= P(1+i)^n \\ 88,673.91 &= P(1+0.04)^{44} \\ (88,673.91)(1.04)^{-44} &= P \\ 15,788.06 &= P \end{aligned}$$

The purchasing power is therefore \$ 15,788.06, and that is something altogether different—but still enough for a nice used car.

To be very clear, the first value of \$ 88,673.91 is what will be in the investment account when Rich retires. The second value of \$ 15,788.06 is the purchasing power of that money, in today's dollars. In other words, a large shopping cart with goods costing \$ 15,788.06 in today's dollars would be expected to cost \$ 88,673.91 when Rich retires.

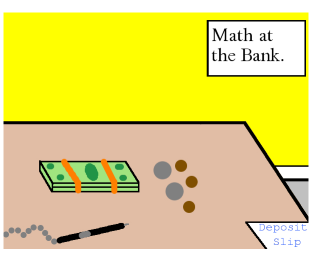
When you are forecasting inflation, what percentage should you use? I read a lot of competing books when writing this textbook. Usually either 3.5% is recommended, or it is recommended that you consider 3% and 4% as strict upper and lower bounds. Some books even say to just use 3%. So I decided to perform a calculation: over the entire span of time that the CPI has existed, what has the inflation done? I took a sample worker, whose career runs from age 21 to age 65. That is $65 - 21 = 44$ elapsed years.

In particular, I can easily find the CPI at the end of their career, and I can also easily find the CPI at the start. The "average rate" over that period of time is actually given by

$$\sqrt[44]{\frac{\text{End CPI}}{\text{Start CPI}}} - 1$$

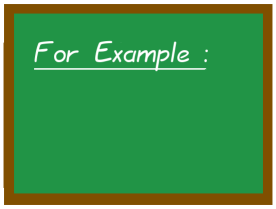
which is probably startling to you. You might be surprised to see the 44th root being used there. This formula will be explained on Page 467. For now, let's just look at what the data tells us. The data is below:

Start Age 21	End Age 65	Start CPI	End CPI	Relative Change	44th Root	Inflation	Start Age 21	End Age 65	Start CPI	End CPI	Relative Change	44th Root	Inflation
1913	1957	9.8	28.1	2.86735	1.02423	2.42%	1940	1984	14.1	103.7	7.35461	1.04639	4.64%
1914	1958	9.9	28.9	2.91919	1.02465	2.47%	1941	1985	14.7	107.6	7.31973	1.04628	4.63%
1915	1959	10.1	29.1	2.88119	1.02434	2.43%	1942	1986	16.3	109.5	6.71779	1.04424	4.42%
1916	1960	10.8	29.6	2.74074	1.02318	2.32%	1943	1987	17.5	113.5	6.48571	1.04341	4.34%
1917	1961	13.0	29.8	2.29231	1.01903	1.90%	1944	1988	17.6	118.0	6.70455	1.04419	4.42%
1918	1962	14.7	30.2	2.05442	1.01650	1.65%	1945	1989	18.1	124.1	6.85635	1.04473	4.47%
1919	1963	16.9	30.6	1.81065	1.01358	1.36%	1946	1990	18.7	129.9	6.94652	1.04504	4.50%
1920	1964	20.9	31.0	1.48325	1.00900	0.90%	1947	1991	22.0	136.0	6.18182	1.04227	4.23%
1921	1965	17.6	31.6	1.79545	1.01339	1.34%	1948	1992	24.1	140.2	5.81743	1.04083	4.08%
1922	1966	16.7	32.4	1.94012	1.01518	1.52%	1949	1993	23.9	144.4	6.04184	1.04173	4.17%
1923	1967	17.0	33.3	1.95882	1.01540	1.54%	1950	1994	23.8	148.0	6.21849	1.04241	4.24%
1924	1968	17.0	34.7	2.04118	1.01635	1.64%	1951	1995	25.9	152.5	5.88803	1.04112	4.11%
1925	1969	17.5	36.6	2.09143	1.01691	1.69%	1952	1996	26.5	156.7	5.91321	1.04122	4.12%
1926	1970	17.7	38.8	2.19209	1.01800	1.80%	1953	1997	26.8	160.3	5.98134	1.04149	4.15%
1927	1971	17.6	40.6	2.30682	1.01918	1.92%	1954	1998	26.9	163.0	6.05948	1.04180	4.18%
1928	1972	17.1	41.7	2.43860	1.02047	2.05%	1955	1999	26.7	166.2	6.22472	1.04243	4.24%
1929	1973	17.1	44.2	2.58480	1.02182	2.18%	1956	2000	27.2	172.4	6.33824	1.04286	4.29%
1930	1974	16.8	49.0	2.91667	1.02463	2.46%	1957	2001	28.1	178.0	6.33452	1.04285	4.29%
1931	1975	15.1	53.6	3.54967	1.02921	2.92%	1958	2002	28.9	179.9	6.22491	1.04243	4.24%
1932	1976	13.6	56.8	4.17647	1.03302	3.30%	1959	2003	29.1	183.7	6.31271	1.04277	4.28%
1933	1977	12.7	60.7	4.77953	1.03619	3.62%	1960	2004	29.6	189.7	6.40878	1.04312	4.31%
1934	1978	13.4	65.2	4.86567	1.03661	3.66%	1961	2005	29.8	194.5	6.52685	1.04356	4.36%
1935	1979	13.7	72.3	5.27737	1.03853	3.85%	1962	2006	30.2	202.9	6.71854	1.04424	4.42%
1936	1980	13.8	82.7	5.99275	1.04153	4.15%	1963	2007	30.6	208.3	6.80719	1.04456	4.46%
1937	1981	14.4	90.6	6.29167	1.04269	4.27%	1964	2008	31.0	218.8	7.05806	1.04541	4.54%
1938	1982	14.1	97.0	6.87943	1.04481	4.48%	1965	2009	31.6	215.6	6.82278	1.04461	4.46%
1939	1983	13.8	99.5	7.21014	1.04592	4.59%							



What is obvious is that in the post-World War II era, the rate is remarkably predictable. The median is 4.26% and half of the "lifetimes" have between 4.18% and 4.39%. First, we can conclude that over a lifetime, inflation is not some unpredictable force. In fact, it is remarkably predictable. Second, it seems that the conventional wisdom of using 3% and 4% is utterly wrong, and instead you should check between 4% and 4.5%.

In particular, the most recent career with an inflation below 4% was someone who turned 21 in 1935, which would be someone who is 100 years old in 2014.

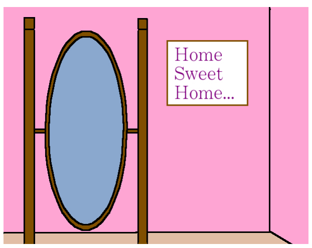


3-6-14

Let's return to the boat problem, which I introduced on Page 459. Based on the data from the large table that we just saw, I'm going to use the higher of the two estimates, namely \$ 30,416, which came from 4% inflation. Suppose my bank is offering a money market account at a rate of 5%, compounded annually. Thus $r = 0.05$ and $i = 0.05$ as well (because $m = 1$), and likewise $n = t = 5$, because I am looking 5 years out. We can compute

$$\begin{aligned} A &= P(1+i)^n \\ 30,416 &= P(1+0.05)^5 \\ 30,416 &= P(1.27628\cdots) \\ \frac{30,416}{1.27628\cdots} &= P \\ 23,831.73 &= P \end{aligned}$$

Does this look a bit off? Let's discuss that in the next box.



A Pause for Reflection...
I could buy the boat now (not five years from now) for \$ 25,000. Or, I could deposit \$ 23,831.73 now, and buy the boat five years from now?

What is going on here? Basically, I get no advantage for saving up for 5 years. More precisely, by waiting 5 years, I only save \$ 1168.27. How can that be? What happened to the "power" of compound interest?!

As it comes to pass, the reason that saving up was totally ineffective is because the rate of inflation (4%) was nearly equal to the interest rate of the investment (5%).

There is a more precise way to understand this. Of course, one can see that if inflation is 4%, and this investment is making 5%, then the investment is barely making headway. In fact, it might be easy to assume that it really is performing at $5\% - 4\% = 1\%$, but that is inaccurate. It would be an over-simplification to assume that.

Each year the money in the bank account is multiplied by 1.05, to reflect the compound interest, and divided by 1.04, to reflect inflation. Thus I have

$$\frac{P(1.05)(1.05)(1.05)(1.05)(1.05)}{(1.04)(1.04)(1.04)(1.04)(1.04)} = P\frac{(1.05)^5}{(1.04)^5} = P\left(\frac{1.05}{1.04}\right)^5 = P(1.00961\cdots)^5 = P(1+0.00961538\cdots)^5$$

Therefore, it is as if I had a compound interest investment that was operating at $0.961538\cdots\% \approx 0.96\%$, compounded annually. Again, that is not far from 1%, but it definitely is not 1%.

Yet another way to look at this is to note that $A/(1.04)^5$ is the purchasing power of A after adjusting for inflation, and $A = P(1.05)^5$. Plugging one formula into the other, then we have

$$A = P\frac{(1.05)^5}{(1.04)^5} = P(1.05/1.04)^5 = P(1.00961\cdots)^5 = P(1+0.00961538\cdots)^5$$

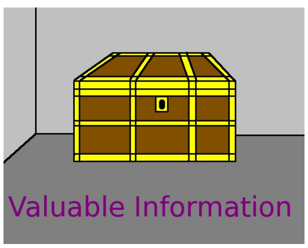
Let's check our work. A deposit of \$ 23,831.73, invested at $0.961538\cdots\%$, compounded annually, for 5 years, would yield

$$A = (23,831.73)(1+0.00961538)^5 = (23,831.73)(1.04901\cdots) = 24,999.73$$

Since we were expecting \$ 25,000, that's quite close enough. On the other hand, a naïve person would say that 5% interest with 4% inflation is 1% interest. We can check to see how that does, by computing

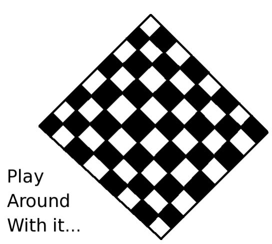
$$A = (23,831.73)(1+0.01)^5 = (23,831.73)(1.05101\cdots) = 25,047.38$$

which is off by \$ 47.38. That's certainly not terrible, but it doesn't meet our benchmark of six significant figures.



So far, we've learned that if you have a rate of interest r_{intr} and a rate of inflation r_{infl}, then

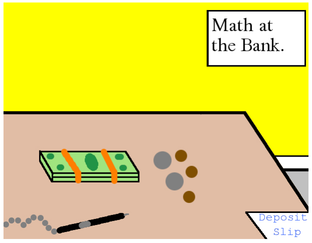
- You can use the compound interest formula once, using r_{intr}, and then adjust for inflation as a second step. That can be done via compound interest formula again, or simply by multiplying by $(r_{infl})^{-t}$.
- You cannot simply assume that the purchasing power is growing at a rate equal to $r_{intr} - r_{infl}$. However, it is not a terrible estimate, in case you want to check your work.
- You might also want a faster way of solving these problems. It turns out that we can do this by computing r_{true}, the *true rate of return*, which is sometimes called the *real rate of return*. This is done with Fisher's equation, which we will now explore.
- Of course, all this applies to problems taking place in the future. For problems in the past, you can use the consumer price index.



3-6-15

Perhaps I have a certificate of deposit (a CD) that is giving 3.5% compounded annually for 3 years when inflation is around 1.5%. Let me deposit \$ 10,000 at the start, and not withdraw anything until the end. Furthermore, I will assume that inflation will remain at 1.5%.

- How much money will I have in the CD at the end? [Answer: \$ 11,087.17.]
- How much is that in today's money? (In other words, in the money of the time of the start of the problem?) [Answer: \$ 10,602.84.]
- How about if, instead, I put the money under my mattress for three years. What then is the value of that money at the end, in today's dollars (i.e. in the money of three years ago)? [Answer: \$ 9563.17.]
- According to Fisher's Equation, what is the real rate of return of the certificate of deposit? [Answer: $1.97044\cdots\%$.]



Looking at the previous box, it becomes clear that the true purpose of very-low-risk but low-return investments is to resist the natural force of inflation. By resisting inflation, I refer to the \$ 10,000 in the CD which grew to \$ 10,602 rather than the \$ 10,000 under the mattress which shrank to \$ 9563.

Basically, you can choose. On the one hand, your \$ 10,000 can evaporate slowly, leaving you with \$ 9563.17. On the other hand, you can invest it so that the \$ 10,000 grows into what is nominally 11,087.17, but which has a purchasing power of \$ 10,602.84.

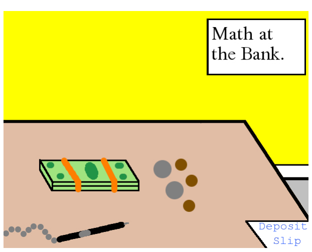
There's another interesting point about the previous checkerboard box. A naïve person might imagine that +3.5% is the growth rate, and −1.5% is the "effect" of inflation, thus 3.5% − 1.5% = 2% is the "real rate." Yet, that's not quite true. If \$ 10,000 grows for three years at 2% then it would be worth

$$A = P(1+i)^n = 10,000(1+0.02)^3 = 10,612.08$$

and that is \$ 9.24 off the correct answer that we obtained earlier. This is a significant error, roughly 1:1000, or a tenth of a percent.

We certainly can't tolerate that kind of error. However, it does also go to show that sometimes, $r_{intr} - r_{infl}$ can serve as a quick-and-easy estimate for situations where you want to either calculate very quickly or perhaps check your work.

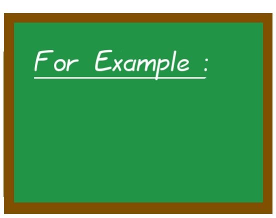
In other words, the answer that Fisher's Equation gave us was $1.97044\cdots\%$, which we can trust because it is close to the naïve estimate of 3.5% − 1.5% = 2%.



The previous checkerboard box is a great opportunity to explain another interpretation of the real rate of return. Observe,

$$A = P(1+i)^n = 10,000(1+0.0197044)^3 = 10,602.85$$

This means that the real rate of return is like an express train, that takes you directly from the principal to the future purchasing power of the final amount, without the need for those intermediate steps. As you can see, in this case we have an answer that is only off by a penny—that is to say, we are only off in the seventh significant figure.



3-6-16

Believe it or not, we can also calculate the real rate of return of stuffing money in a mattress. The nominal rate, of course, is 0%. The hundred-dollar bills are inanimate objects—they will neither die nor breed with each other. Therefore, the number of them remains unchanged over the three years that they are hidden in the mattress. The rate of inflation was 1.5%.

Using Fisher's Equation, we have

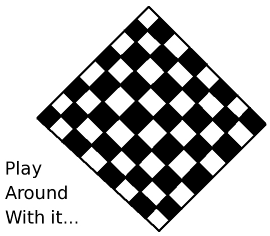
$$r_{real} = \frac{r_{intr} - r_{infl}}{1 + r_{infl}} = \frac{0 - 0.015}{1.015} = -0.0147783\cdots$$

Fisher's Equation is telling us that the real rate of return is negative $1.47783\cdots\%$. We will check this in the next box.

It should make sense that the real rate of return in the previous box was negative, because I am losing purchasing power to inflation if I just stuff cash into my mattress.

$$A = P(1+i)^n = 10,000(1-0.0147783)^3 = 9563.17$$

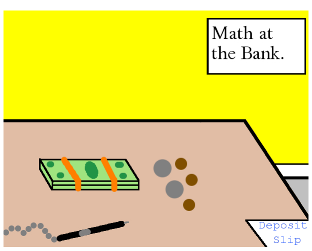
What is awesome, at least in my opinion, is that the real rate of return computes precisely and exactly what the purchasing power turns into. It really is an express train connecting the starting principal with the ending purchasing power. As you can see, in this case, we were exact to the penny!



3-6-17

Let's repeat the above analysis with a Certificate of Deposit, again at 3.5% compounded annually. However, let's see what happens if inflation turned out not to stay at 1.5%, but instead became 2%.

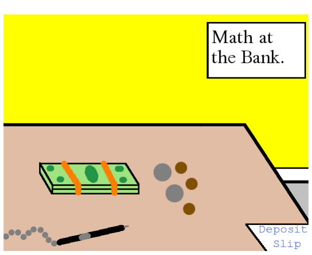
- What is in the CD at the end? [Answer: \$ 11,087.17.]
- What is that in "today's dollars"? [Answer: \$ 10,447.68.]
- What is the value of the money under the mattress in "today's dollars"? [Answer: \$ 9423.22.]
- What is the real rate of return of the CD? [Answer: $+1.47058\cdots\%$.]
- What is the real rate of return of the money stuffed into the mattress? [Answer: $-1.96078\cdots\%$.]



To make a grand summary of the discussion, the previous box gives me two options.

- Using the CD, I finish with \$ 11,087.17 instead of \$ 10,000 with the mattress, a difference of roughly a grand.
- Using the CD, I have a purchasing power of \$ 10,447.68 instead of \$ 9423.22 with the mattress, again, a difference of roughly a grand.
- Using the CD, my money earned +1.47% instead of −1.96% with the mattress. Being on opposite sides of zero, those two rates are obviously rather different from each other.

When we save, we use banks not so much because our purchasing power will grow. (In this case, it only would grow by \$ 447.68.) Instead, we use banks because if we didn't, our purchasing power would slowly evaporate.



The Consumer Price Index is not the only measure of inflation. There is also the *Producer Price Index* or PPI. Its purpose is to measure the costs of producing items domestically, and is not strictly restricted to manufacturing in a strict sense, but includes tasks like the construction of buildings and home improvement. The PPI includes in its tabulation things that producers might buy, but which consumers like you and I might not, such as large pieces of factory equipment. Also, the PPI would exclude things like haircuts and doctor's visits, which would be included in the CPI. Thus the two indices are not all that closely matched.

Any domestically produced item has to be produced before it can be consumed. Usually some period of time will elapse, including the manufacturing process, shipping to a wholesaler, shipping to a retailer, sitting on a shelf, and finally ending with purchase. Even with internet shopping, the item would sit in a warehouse waiting to be bought. With this in mind, despite the differences in the kind of items covered by the indices, some economists use the PPI as a predictor of the CPI, with the idea that an increase in the PPI would result in items being more expensive to produce, and that in turn should drive up prices at the consumer side. This practice is expressly discouraged by the Bureau of Labor Statistics, however. Far too many other variables come into play.

The topic of the previous box is an example of how mathematical models can be abused. One thing is said to predict another, and good mathematics might be written down after that assumption. Yet if the underlying assumption (here, that the PPI predicts the CPI) is even sometimes false, then the resulting predictions could quite possibly be false, even if the mathematics is perfect. It is also an example of how something accepted by the financial community as a "standard practice" might well be standard, but nonetheless unwise or misleading.

If the PPI does not predict the CPI well, then any mathematics based on the assumption that the PPI predicts the CPI will produce unreliable answers, even if the mathematics contains no errors.

You can read more about the PPI and comparisons to the CPI at `http://www.bls.gov/ppi/ppicpippi.htm`

I still owe you an explanation of the formula

$$\sqrt[44]{\frac{\text{End CPI}}{\text{Start CPI}}} - 1$$

which you saw earlier on Page 462.

You cannot take 44 separate CPIs and just average them, in the sense of adding them all up and dividing by 44. To explain why you cannot is a bit tedious, and involves something called the "arithmetic mean" versus the "geometric mean."

In any case, let's imagine a compound interest problem, where the principal P is the "Start CPI" and it grows into the "End CPI," which will be our amount A. This occurs over 44 years, and inflation is computed by compounding annually. The rate is the rate of inflation, r_{infl}. In other words, we are modeling the inflation by imagining the CPI starting at one value, and slowly but steadily inflating into the ending value. We have, therefore

$$\begin{aligned}
A &= P(1+i)^n \\
(\text{End CPI}) &= (\text{Start CPI})(1+r_{infl})^{44} \\
\frac{\text{End CPI}}{\text{Start CPI}} &= (1+r_{infl})^{44} \\
\sqrt[44]{\frac{\text{End CPI}}{\text{Start CPI}}} &= 1+r_{infl} \\
\sqrt[44]{\frac{\text{End CPI}}{\text{Start CPI}}} - 1 &= r_{infl}
\end{aligned}$$

That's where our strange looking formula came from.

We're now going to explore how inflation can save us from embarrassing misunderstandings of historical data. Suppose I look at a chart of how the minimum wage changed historically. This is given in the large table below as "Actual Min. Wage." As you can see, the minimum wage is increasing over time. Because it is increasing over time, I might suspect that the condition of a minimum-wage worker in, perhaps in 1978, would be much less well off than one in 1995. However, the data shows that this notion is *extremely* mistaken.

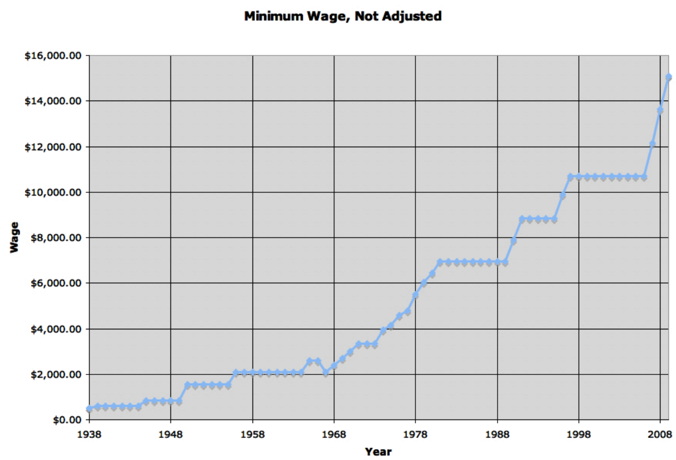
The historical minimum wages came from a paper from Oregon State University's Department of Anthropology. Next, I then took the data on the CPI given earlier, and one-by-one adjusted each year's value. This is the column "Adjusted Minimum Wage." Finally, both columns were multiplied by 40 and 52, to represent working 40 hours per week and 52 weeks per year.

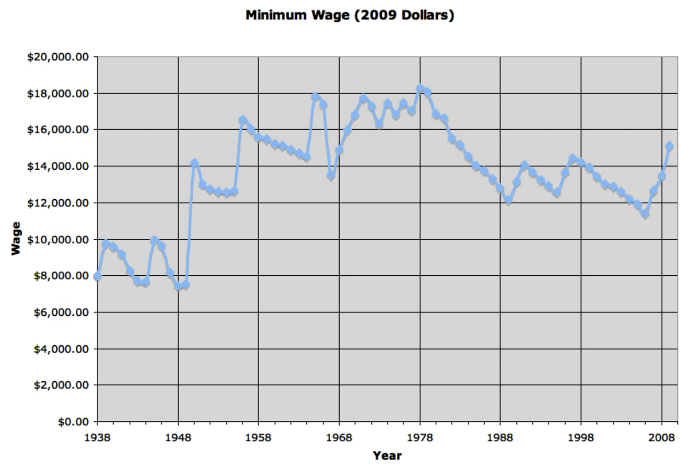
Okay, now look at 1978: in 2009 dollars, our minimum-wage worker would be earning \$ 18,323.26, but the 1995 minimum-wage worker would be earning (in 2009 dollars) a salary of \$ 12,563.87. What a huge difference inflation makes!

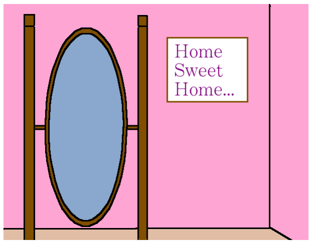
Year	Actual Min. Wage	Adjusted Min. Wage	40-hours per week Not Adj'd.	40-hours per week Adjusted	Year	Actual Min. Wage	Adjusted Min. Wage	40-hours per week Not Adj'd.	40-hours per week Adjusted
1938	\$ 0.25	\$ 3.84	\$ 520.00	\$ 7,993.29	1974	\$ 1.90	\$ 8.40	\$ 3,952.00	\$ 17,480.83
1939	\$ 0.30	\$ 4.71	\$ 624.00	\$ 9,800.46	1975	\$ 2.00	\$ 8.09	\$ 4,160.00	\$ 16,821.69
1940	\$ 0.30	\$ 4.61	\$ 624.00	\$ 9,591.94	1976	\$ 2.20	\$ 8.39	\$ 4,576.00	\$ 17,461.39
1941	\$ 0.30	\$ 4.42	\$ 624.00	\$ 9,200.43	1977	\$ 2.30	\$ 8.21	\$ 4,784.00	\$ 17,082.19
1942	\$ 0.30	\$ 3.99	\$ 624.00	\$ 8,297.32	1978	\$ 2.65	\$ 8.81	\$ 5,512.00	\$ 18,323.26
1943	\$ 0.30	\$ 3.72	\$ 624.00	\$ 7,728.36	1979	\$ 2.90	\$ 8.69	\$ 6,032.00	\$ 18,082.73
1944	\$ 0.30	\$ 3.69	\$ 624.00	\$ 7,684.45	1980	\$ 3.10	\$ 8.12	\$ 6,448.00	\$ 16,898.98
1945	\$ 0.40	\$ 4.79	\$ 832.00	\$ 9,962.90	1981	\$ 3.35	\$ 8.01	\$ 6,968.00	\$ 16,669.44
1946	\$ 0.40	\$ 4.64	\$ 832.00	\$ 9,643.24	1982	\$ 3.35	\$ 7.49	\$ 6,968.00	\$ 15,569.60
1947	\$ 0.40	\$ 3.94	\$ 832.00	\$ 8,196.75	1983	\$ 3.35	\$ 7.30	\$ 6,968.00	\$ 15,178.40
1948	\$ 0.40	\$ 3.60	\$ 832.00	\$ 7,482.51	1984	\$ 3.35	\$ 7.00	\$ 6,968.00	\$ 14,563.66
1949	\$ 0.40	\$ 3.63	\$ 832.00	\$ 7,545.13	1985	\$ 3.35	\$ 6.75	\$ 6,968.00	\$ 14,035.79
1950	\$ 0.75	\$ 6.83	\$ 1,560.00	\$ 14,206.55	1986	\$ 3.35	\$ 6.63	\$ 6,968.00	\$ 13,792.25
1951	\$ 0.75	\$ 6.28	\$ 1,560.00	\$ 13,054.67	1987	\$ 3.35	\$ 6.40	\$ 6,968.00	\$ 13,306.18
1952	\$ 0.75	\$ 6.13	\$ 1,560.00	\$ 12,759.09	1988	\$ 3.35	\$ 6.15	\$ 6,968.00	\$ 12,798.74
1953	\$ 0.75	\$ 6.07	\$ 1,560.00	\$ 12,616.27	1989	\$ 3.35	\$ 5.85	\$ 6,968.00	\$ 12,169.63
1954	\$ 0.75	\$ 6.04	\$ 1,560.00	\$ 12,569.37	1990	\$ 3.80	\$ 6.34	\$ 7,904.00	\$ 13,188.00
1955	\$ 0.75	\$ 6.09	\$ 1,560.00	\$ 12,663.52	1991	\$ 4.25	\$ 6.77	\$ 8,840.00	\$ 14,088.17
1956	\$ 1.00	\$ 7.97	\$ 2,080.00	\$ 16,574.31	1992	\$ 4.25	\$ 6.57	\$ 8,840.00	\$ 13,666.12
1957	\$ 1.00	\$ 7.71	\$ 2,080.00	\$ 16,043.46	1993	\$ 4.25	\$ 6.38	\$ 8,840.00	\$ 13,268.63
1958	\$ 1.00	\$ 7.50	\$ 2,080.00	\$ 15,599.35	1994	\$ 4.25	\$ 6.22	\$ 8,840.00	\$ 12,945.88
1959	\$ 1.00	\$ 7.45	\$ 2,080.00	\$ 15,492.14	1995	\$ 4.25	\$ 6.04	\$ 8,840.00	\$ 12,563.87
1960	\$ 1.00	\$ 7.32	\$ 2,080.00	\$ 15,230.45	1996	\$ 4.75	\$ 6.57	\$ 9,880.00	\$ 13,665.61
1961	\$ 1.00	\$ 7.27	\$ 2,080.00	\$ 15,128.23	1997	\$ 5.15	\$ 6.96	\$ 10,712.00	\$ 14,483.65
1962	\$ 1.00	\$ 7.18	\$ 2,080.00	\$ 14,927.86	1998	\$ 5.15	\$ 6.85	\$ 10,712.00	\$ 14,243.74
1963	\$ 1.00	\$ 7.08	\$ 2,080.00	\$ 14,732.72	1999	\$ 5.15	\$ 6.72	\$ 10,712.00	\$ 13,969.49
1964	\$ 1.00	\$ 6.99	\$ 2,080.00	\$ 14,542.62	2000	\$ 5.15	\$ 6.47	\$ 10,712.00	\$ 13,467.11
1965	\$ 1.25	\$ 8.57	\$ 2,600.00	\$ 17,833.12	2001	\$ 5.15	\$ 6.27	\$ 10,712.00	\$ 13,043.42
1966	\$ 1.25	\$ 8.36	\$ 2,600.00	\$ 17,392.80	2002	\$ 5.15	\$ 6.20	\$ 10,712.00	\$ 12,905.67
1967	\$ 1.00	\$ 6.51	\$ 2,080.00	\$ 13,538.18	2003	\$ 5.15	\$ 6.08	\$ 10,712.00	\$ 12,638.70
1968	\$ 1.15	\$ 7.18	\$ 2,392.00	\$ 14,940.76	2004	\$ 5.15	\$ 5.88	\$ 10,712.00	\$ 12,238.95
1969	\$ 1.30	\$ 7.70	\$ 2,704.00	\$ 16,012.78	2005	\$ 5.15	\$ 5.74	\$ 10,712.00	\$ 11,936.91
1970	\$ 1.45	\$ 8.10	\$ 3,016.00	\$ 16,847.70	2006	\$ 5.15	\$ 5.50	\$ 10,712.00	\$ 11,442.73
1971	\$ 1.60	\$ 8.54	\$ 3,328.00	\$ 17,766.36	2007	\$ 5.85	\$ 6.09	\$ 12,168.00	\$ 12,657.93
1972	\$ 1.60	\$ 8.32	\$ 3,328.00	\$ 17,297.70	2008	\$ 6.55	\$ 6.49	\$ 13,624.00	\$ 13,494.87
1973	\$ 1.60	\$ 7.85	\$ 3,328.00	\$ 16,319.32	2009	\$ 7.25	\$ 7.29	\$ 15,080.00	\$ 15,153.27

In this table, "adjusted for inflation" refers to the dollars of the year 2010.

The raw data can be found at `http://oregonstate.edu/instruct/anth484/minwage.html`

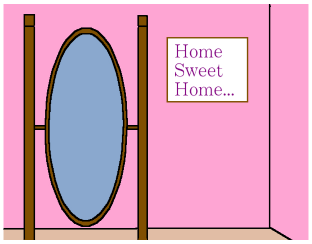






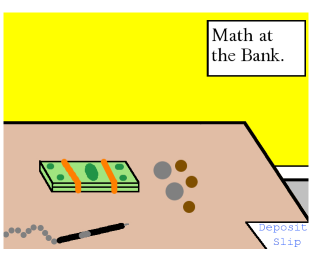
A Pause for Reflection...
Above, you have two graphs, one showing the annual minimum wage after adjusting for inflation (on the right) and the other not adjusting for it (on the left). Each has its own lies to tell.

As I mentioned before, you might imagine that the lifestyle of the minimum-wage worker is strictly continuously improving when you look at the graph on the left. But, as we showed before, this is not the case. The difference between an annual salary of $ 18,323.26 (in 1978) and of $ 12,563.87 (in 1995) is, in terms of lifestyle, huge.

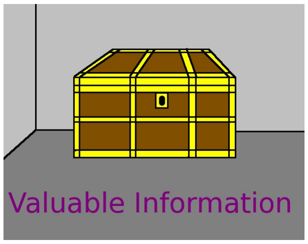


A Pause for Reflection...
However, the graph on the right is also misleading. You would imagine, looking only at it, that the minimum wage was changing constantly.

As you can see from looking at the graph on the left, there were long periods where the wage was totally left unchanged. The minimum wage is set by law, and so if the old law is not repealed by enacting a new law, then the minimum wage remains at a fixed value.



The moral of the story is that, when analyzing historical data for a business report or a class project, whether in academia or in "the real world," you should always present both the adjusted and the non-adjusted. Preferably you should supply not only a graph, but make the raw data (i.e. the numbers and not just a mere graph) available somehow, either as an appendix or on your website, so that future researchers can use it. I got my data for those charts, after all, from just such a web page.



Suppose I have an investment paying r_{intr} in interest, compounded annually, but that inflation is at the rate r_{infl}. These forces are working in opposite directions. It would be very useful to find the "net effect." What this would be is the growth in the purchasing power, in contrast with the growth of the number printed on the account statement, which is not as important. In other words, I want to know the growth rate after adjusting for inflation. That rate is called r_{true}, the *true rate of interest*, or sometimes the *real rate of return*.

The formula for r_{true} (sometimes called r_{real}) is

$$r_{true} = \frac{r_{intr} - r_{infl}}{1 + r_{infl}}$$

and this is called The Fisher Equation, named for Irving Fisher. It is sometimes equivalently written

$$(1 + r_{infl})(1 + r_{true}) = (1 + r_{intr})$$

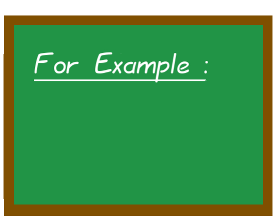
You can think of r_{true} as how much interest the investment is paying me in constant purchasing power, or in terms of the dollars of the starting year. The r_{intr} is in terms of actual dollars on the account statement, or dollars of the ending year.

Very shortly, we will begin a rigorous theoretical derivation of Fisher's Equation. However, there's a silly explanation that I like a great deal, but it is a bit awkward to explain. We will consider two scenarios, on two planets. One is our planet, and it has inflation. Then there is an inflation-free planet. (Even if you do not like science fiction, just bare with me for this one box.)

If I invested on our planet, with inflation r_{infl} and interest rate r_{intr}, then I get some amount A. It has slightly reduced purchasing power compared to what I expect, because of inflation.

By r_{true}, the *true rate of interest*, what we mean is that if I had invested instead on the inflation-free planet, but not at rate r_{intr} but instead at r_{true}, then I'd end with A as well. Of course, that A does not have its purchasing power reduced by inflation, because it is on the inflation-free planet.

The Fisher equation tells you how to find the value of r_{true} so that the A on each planet is the same dollar amount.

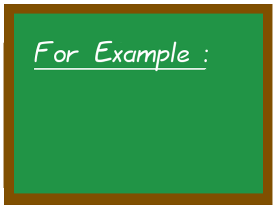


Suppose I see an ad for a certificate of deposit yielding 4.5%, and I know that inflation is currently 2.25%. What is the true rate of return?

$$r_{true} = \frac{r_{intr} - r_{infl}}{1 + r_{infl}} = \frac{0.045 - 0.0225}{1 + 0.0225} = \frac{0.0225}{1.0225} = 0.0220048\cdots$$

So we can see that it is about 2.20%.

3-6-18



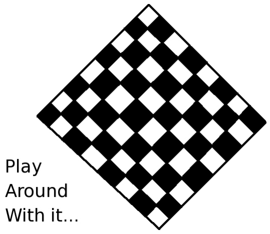
If the anticipated rate of return is 8% on a particular mutual fund, but inflation is predicted to be 3.5%, what is the true rate of return?

Using the formula in the previous box, we would have

$$r_{true} = \frac{r_{intr} - r_{infl}}{1 + r_{infl}} = \frac{0.08 - 0.035}{1 + 0.035} = \frac{0.045}{1.035} = 0.0434782\cdots$$

we conclude that the true rate of return is $4.34782\cdots\% \approx 4.34\%$.

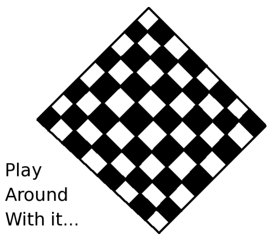
3-6-19



How about a mutual fund with 9% on a particular rate of return? If we assume that inflation will be between 3% and 4%, then what is the interval or range of the possible true rates of interest? Round to the nearest basis point.

[Answer: between 5.83% and 4.81%.]

3-6-20



During the height of the financial crisis of 2008–2009, a bank near me was offering a 1% interest rate on a certificate of deposit. But, the rate of inflation at that point was about -0.5% (that is to say, negative half a percent). This is called deflation, and it can happen during bad economies (and, indeed, it did). What is the "true rate of return" of this certificate? [Answer: $1.50753\cdots\%$.]

3-6-21

An approximation

$$r_{true} \approx r_{intr} - r_{infl}$$

can be used to check your work. It can also be used in a situation when a calculator is not handy, to give a very rough approximation. We saw the following examples, in the previous four boxes:

- Interest of 4.5% and inflation of 2.25% gives a true rate of 2.20%, but $4.5\% - 2.25\% = 2.25\%$.
- Interest of 8% and inflation of 3.5% give a true rate of 4.34%, but $8\% - 3.5\% = 4.5\%$.
- Interest of 9% and inflation of 3% give a true rate of 5.83%, but $9\% - 3\% = 6\%$.
- Interest of 9% and inflation of 4% give a true rate of 4.81%, but $9\% - 4\% = 5\%$.
- Interest of 1% and inflation of -0.5% give a true rate of 1.50753%, but $1\% - (-0.5\%) = 1.50000\%$.

In the first case and the last case, the approximation was relatively okay, but in the other cases it was off significantly. Nonetheless, it can be useful in situations when a calculator is not nearby.

Now we will derive Fisher's Equation, in the next three boxes.

- We are going to consider interest at rate of r_{intr} compounded annually, under a rate of inflation r_{infl}, and try to determine the true rate of return. Let P denote the principal, and A denote the amount.
- Over t years, an investment at the interest rate r_{intr} will grow from P to A according to the normal compound interest formula $A = P(1+i)^n$.
- Since we are compounding annually, we can write $A = P(1+r_{intr})^t$.
- On Page 459, we saw that the purchasing power of x dollars, t years in the future, with rate of inflation r_{infl} will be $x(1+r_{infl})^{-t}$.
- Therefore, the purchasing power of the final amount A is given by $A(1+r_{infl})^{-t}$.
- The purchasing power of P dollars at the start is P dollars. Therefore, we want to know what rate r_{true} will bring us from the current purchasing power P to the future purchasing power $A(1+r_{infl})^{-t}$. Let $B = A(1+r_{infl})^{-t}$, to simplify the notation.
- We can summarize by saying that we want a true rate of return that takes us from P to B. Of course, this means $B = P(1+r_{true})^t$.

The facts in the previous box are enough to complete the derivation with mere algebra.

$$\begin{aligned} B &= P(1+r_{true})^t \\ A(1+r_{infl})^{-t} &= P(1+r_{true})^t \\ A &= P(1+r_{true})^t(1+r_{infl})^t \\ P(1+r_{intr})^t &= P(1+r_{true})^t(1+r_{infl})^t \\ (1+r_{intr})^t &= (1+r_{true})^t(1+r_{infl})^t \end{aligned}$$

Finally, we take the "t-th root" of both sides and get one form of Fisher's Equation:

$$(1+r_{intr}) = (1+r_{true})(1+r_{infl})$$

Continuing from the previous box, we can derive the other form now:

$$\begin{aligned} (1+r_{true})(1+r_{infl}) &= 1+r_{intr} \\ 1+r_{true} &= \frac{1+r_{intr}}{1+r_{infl}} \\ r_{true} &= \frac{1+r_{intr}}{1+r_{infl}} - 1 \\ r_{true} &= \frac{1+r_{intr}}{1+r_{infl}} - \frac{1+r_{infl}}{1+r_{infl}} \\ r_{true} &= \frac{1+r_{intr}-1-r_{infl}}{1+r_{infl}} \\ r_{true} &= \frac{r_{intr}-r_{infl}}{1+r_{infl}} \end{aligned}$$

I think most students find the form given in the previous box easier to remember, and the form given in this box to be easier to use.

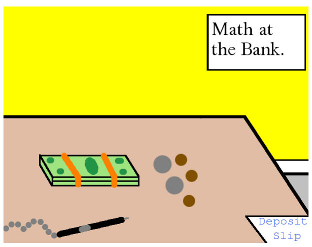
The Fisher equation is named for Irving Fisher (1867–1947), an American economist who was very famous in his time. In 1891, he was the very first person to receive a Ph.D. in Economics from Yale, an amazing three years after his Bachelor's degree in 1888. He is said to have paid his way through Yale, while supporting his widowed mother, by tutoring mathematics. Among many economic accomplishments, he is known for trying to bring mathematical sophistication to the subject of economics.

As mentioned before, the common way of writing the Fisher Equation is

$$1 + r_{true} = (1 + r_{intr})(1 + r_{infl})$$

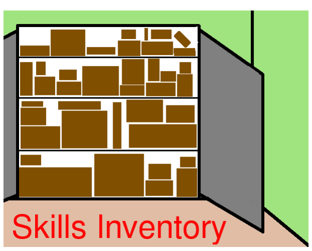
and just as Einstein's Theory of relativity is far more than just the sequence of symbols $E = mc^2$, it should be noted that Fisher's dissertation contained a great deal more than just this formula.

For example, there was an excellent discussion of how the rates of inflation in two countries (with large, fully developed economies) would influence each other's interest rates. This more complex problem is outside the scope of this book, but has been very influential in developing modern theories of international trade.



The US Treasury offers inflation-indexed bonds, which return some stated rate *above inflation*, as calculated by the Consumer Price Index. The stated rate can vary, but it is typically between 1.5% and 3.5%. These are called TIPS or "Treasury Inflation Protected Securities," or "I-Series" bonds.

These bonds are a unique type of investment, because you can purchase a fixed real rate of return, but the nominal rate might change. That's in stark contrast to an ordinary bond, where you purchase a nominal rate, but the real rate might change with inflation.



We have learned the following skills in this module:

- To use the consumer price index, to convert old financial data into today's dollars.
- To use inflation to analyze historical data, such as the minimum wage.
- To use predicted value of inflation to forecast the decay of money into the future.
- To use Fisher's Equation to convert interest rates and inflation rates into true rates of return.
- As well as the vocabulary terms: consumer price index (CPI), inflation, producer price index (PPI), real rate of return, true rate of interest.