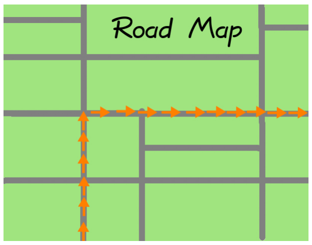
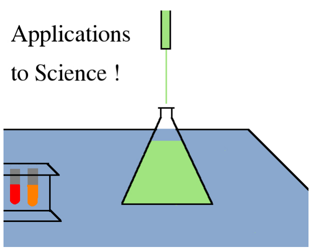
Module A.1: Unit Conversions



Many of the best jobs in the world involve international trade. This is especially true in the USA, because relatively little manufacturing of heavy equipment occurs here. Accordingly, almost all industrial equipment is imported from abroad. As you are already aware, different countries tend to use different units to measure just about everything. (For example, meters versus feet, dollars versus euros, pounds versus kilograms.)

Moreover, sales jobs also typically involve international trade. After all, there are more than seven billion people on the planet, and about 5% of them live in the USA. Most companies cannot afford to ignore 95% of the worlds's population.

While unit conversions can be a bit irritating at times, they are an important skill in the toolbox of a business student. Think of it this way—would you rather take business trips to Gary, Indiana and Detroit, Michigan? Or would you rather take business trips to France and Japan?



Unit conversions are not just about international trade. Even staying strictly within the USA, there are points of confusion.

- At a gas station, the ratings for putting air in the tires of your car use the units of pounds per square inch (psi).
- The weather report for flying recreational aircraft measures pressure in millibars (mbar). However `www.aviationweather.gov` also lists the old-fashioned unit "inches of mercury" (inHg) for elderly fliers.
- When going scuba diving, the usual unit is atmospheres (atm).
- As it turns out, my middle-school chemistry class used atmospheres for pressure also.
- Most high-school chemistry classes use Pascals (Pa) as the unit for pressure.
- Mechanical engineering classes often use pounds per square foot (lb/ft^2) or tons per square foot (ton/ft^2).
- Safety regulations might be in millibars, atmospheres, or "millimeters of mercury" (mmHg).

Clearly, this is insane—we have too many units for measuring pressure! Luckily, we'll be focusing on industrial problems and international commerce for this module.

The method which I am showing you here will make the hardest problems relatively straightforward and approachable. Even medium-level problems become easy and simple. On the one hand, unit conversions are often intimidating or baffling for students (and even for some scientists) when taught intuitively; on the other hand, I know from experience that students who learn the technique in this module can indeed repeatedly and perfectly tackle the most difficult unit conversion problems.

However, some students have pointed out that my method requires a bit too much writing, and therefore a disproportionate amount of time, for easy unit conversions. For example, if someone asks you "How many yards are there in 6000 feet, knowing that a yard is 3 feet?" then you probably don't need the method explained in this module for that problem.

For Example :

A-1-1

Imagine that a younger relative is reading about fighter jets. Repeatedly, the websites or books refer to the speed of sound. Naturally, the young fellow searches on the internet for the speed of sound, to find out what this is all about. One site says that the speed of sound is 331.460 meters per second, but he wants to know how many miles per hour that is. Can you help him figure that out?

Surely we know that 60 seconds make a minute, and 60 minutes make an hour. It turns out that a mile is roughly 1609.34 meters.
Now observe,

$$\begin{aligned}
\frac{331.460 \text{ meters}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1609.34 \text{ meters}} &= \\
\frac{331.460 \text{ meters}}{1 \cancel{\text{ second}}} \times \frac{60 \cancel{\text{ seconds}}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1609.34 \text{ meters}} &= \\
\frac{331.460 \text{ meters}}{1 \cancel{\text{ second}}} \times \frac{60 \cancel{\text{ seconds}}}{1 \cancel{\text{ minute}}} \times \frac{60 \cancel{\text{ minutes}}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1609.34 \text{ meters}} &= \\
\frac{331.460 \cancel{\text{ meters}}}{1 \cancel{\text{ second}}} \times \frac{60 \cancel{\text{ seconds}}}{1 \cancel{\text{ minute}}} \times \frac{60 \cancel{\text{ minutes}}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1609.34 \cancel{\text{ meters}}} &= \\
\frac{331.460 \times 60 \times 60 \times 1 \text{ miles}}{1 \times 1 \times 1 \times 1609.34 \text{ hour}} &= 741.456 \cdots \frac{\text{miles}}{\text{hour}}
\end{aligned}$$

For Example :

A-1-2

Let's suppose that you are traveling with a small delegation from your company to a conference in Germany. Some German competitors are teasing your coworkers for the fuel-inefficiency of American Sport Utility Vehicles (SUVs). To counter this jab, your boss points out that her hybrid, a Toyota Prius, gets 51 miles per gallon. Of course, Germany uses the metric system, so they don't know precisely what that means. In Germany, they use kilometers instead of miles, and they use liters instead of gallons. Can you convert miles per gallon to kilometers per liter? As we saw in the previous problem, 1609.34 meters is one mile. Moreover, a quick internet search reveals that a gallon is 3.78541 liters.

The next box will show how I would approach this problem with my methods.

Continuing with the previous box,

$$\begin{aligned}
\frac{51 \text{ miles}}{1 \text{ gallon}} \times \frac{1609.34 \text{ meters}}{1 \text{ mile}} \times \frac{1 \text{ kilometer}}{1000 \text{ meters}} \times \frac{1 \text{ gallon}}{3.78541 \text{ liters}} &= \\
\frac{51 \cancel{\text{ miles}}}{1 \cancel{\text{ gallon}}} \times \frac{1609.34 \cancel{\text{ meters}}}{1 \cancel{\text{ mile}}} \times \frac{1 \text{ kilometer}}{1000 \cancel{\text{ meters}}} \times \frac{1 \cancel{\text{ gallon}}}{3.78541 \text{ liters}} &= \\
\frac{51 \times 1609.34 \times 1 \times 1 \text{ kilometer}}{1 \times 1 \times 1000 \times 3.78541 \text{ liters}} &= 21.6822 \cdots \text{ kilometers/ Liter}
\end{aligned}$$

Therefore, we can conclude that 51 miles per gallon is $21.6822\cdots$ kilometers/liter. Of course, since we only have two digits of precision on the 51 miles per gallon, then we should probably round 21.6822 to 22 in conversation. However, this textbook is not a conversation with coworkers. It is the convention in this textbook to show six significant digits.

You'll notice that the wording of the problem did not remind you that 1000 meters is one kilometer. I'm sure that almost all my readers know that the "kilo-" prefix means "one thousand." For example, when someone says "Agnes makes 85k a year," all of us know that this means "Agnes makes 85,000 a year." The reason that we say "85k" when speaking with coworkers is because everyone knows that "kilo-" means "one thousand" and saying "k" is just a slang abbreviation for "kilo."

I'd like to take a moment to explain what just happened here. We all know that $6/6 = 1$ and $8/8 = 1$. We can generalize this by saying that any fraction where the numerator and denominator are equal and non-zero will be equal to 1. Now with that in mind,

- Because 1 mile equals 1609.34 meters, then we know that

$$\frac{1 \text{ mile}}{1609.34 \text{ meters}} = 1$$

- Because 1 minute equals 60 seconds, then we know that

$$\frac{1 \text{ minute}}{60 \text{ seconds}} = 1$$

- Because 1 hour equals 60 minutes, then we know that

$$\frac{1 \text{ hour}}{60 \text{ minutes}} = 1$$

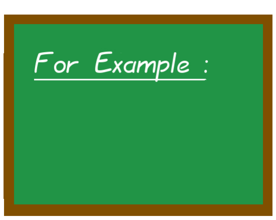
- Because 1 gallon equals 3.78541 liters, then we know that

$$\frac{1 \text{ gallon}}{3.78541 \text{ liters}} = 1$$

As you can see, the only thing that I did in the previous two examples was just multiply by one! Those four fractions listed above are just unusual names for the number one.

What happens when we multiply a number by one? Absolutely nothing! Therefore, the number that I get at the end of my process is equal to the number that I had at the start of the process. I am only multiplying by one. Therefore, we can be absolutely certain that

- 331.460 meters per second = 741.456 miles per hour.
- 51 miles per gallon = 21.6822 kilometers per liter.



A-1-3

Suppose a watch advertisement says that it will survive a pressure of 4 atmospheres. Your friend, who is a marine biologist, will be going undersea as part of an experiment, and the professor running the experiment said that the pressure will reach 300 kilopascals. In order to know if this watch can be loaned to your friend (and survive the diving), you need to know how many atmospheres equal 300 kilopascals. A quick internet search reveals that one atmosphere is 101,325 pascals.

$$\begin{aligned} 300 \text{ kilopascals} \times \frac{1000 \text{ pascals}}{1 \text{ kilopascal}} \times \frac{1 \text{ atmosphere}}{101,325 \text{ pascals}} &= \\ 300 \cancel{\text{ kilopascals}} \times \frac{1000 \cancel{\text{ pascals}}}{1 \cancel{\text{ kilopascal}}} \times \frac{1 \text{ atmosphere}}{101,325 \cancel{\text{ pascals}}} &= \\ \frac{300 \times 1000 \times 1 \text{ atmosphere}}{1 \times 101,325} &= 2.96076\cdots \text{ atmospheres} \end{aligned}$$

Therefore, it seems that the watch will be fine.

For Example :

\# A-1-4

How much would one million dollars, paid in pennies, weigh? (For our foreign students, a penny is the name of the one cent coin.) A quick internet search reveals that, officially, a penny weighs 2.5 grams. Moreover, there are 453.592 g in a pound.

$$
\begin{aligned}
1,000,000 \text{ dollars} \times \frac{100 \text{ pennies}}{1 \text{ dollar}} \times \frac{2.5 \text{ g}}{1 \text{ penny}} \times \frac{1 \text{ lb}}{453.592 \text{ g}} &= \\
1,000,000 \cancel{\text{dollars}} \times \frac{100 \cancel{\text{pennies}}}{1 \cancel{\text{dollar}}} \times \frac{2.5 \cancel{\text{g}}}{1 \cancel{\text{penny}}} \times \frac{1 \text{ lb}}{453.592 \cancel{\text{g}}} &= \\
1,000,000 \times \frac{100 \times 2.5 \times 1}{1 \times 1 \times 453.592} &= 551,156.\cdots \text{ lbs}
\end{aligned}
$$

As you can see, a million dollars in pennies would weigh a lot. It weighs more than half a million pounds. Moreover, we could divide by 2000 and see that this is $275.578\cdots$ tons.

Play Around With it...

\# A-1-5

Let's say that you have just crossed the Atlantic for a business trip in France. Imagine that after hailing a cab to take you from the airport to your hotel, you fall asleep in the cab. You wake up while the cab is speeding along La Peripherique (the big highway that encircles Paris, just like our Capital Beltway encircles Washington, DC). You see that the cab is driving 125, according to the speedometer. At first you are startled, but then you realize that this must mean 125 kph, not 125 mph, as France uses the metric system. (Actually, France is where the metric system was born.) When you finally get to the hotel, you want to know what 125 kph actually means, in terms of mph.

A quick internet search would reveal that there are 1609.34 meters in a mile, and of course, there are 1000 meters in a kilometer. Using this information, tell me what speed, in miles per hour, equals 125 kilometers per hour. [Answer: 77.6715 mph.]

Play Around With it...

\# A-1-6

In the Summer Olympics of 2008, Usain Bolt from Jamaica ran the 100 meter dash in 9.58 seconds, winning the gold metal. As of 2016, this was the world's record, and therefore he is often called "The Fastest Man on Earth." As we've noted before, there are 1609.34 meters in a mile.

What was his average speed in miles per hour? [Answer: $23.3501\cdots$ miles per hour.]

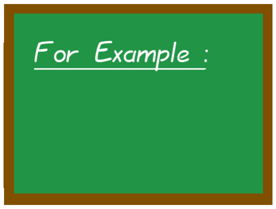
Play Around With it...

\# A-1-7

In the town where I teach (Menomonie, Wisconsin) there is a war monument. The names of those citizens of the town who have fallen in battle are carved neatly into red bricks, forming a pavement around the monument. The bricks are wide, but only 2 inches tall.

Suppose one wanted to build a monument to the six million Jews who had died in the Holocaust, by stacking similar bricks one atop the other, to make a pillar. How tall would such a pillar be in feet? in miles? Recall, there are 5280 feet in a mile, and 12 inches in a foot. [Answer: 1,000,000 feet or $189.393\cdots$ miles.]

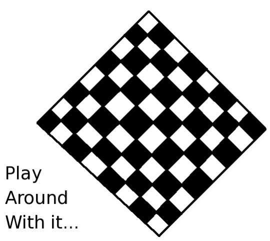
For comparison, Mount Everest is 5.5 miles above sea level. Needless to say, such a pillar would, if constructed, immediately tip over. Another point of comparison might be that the population of Wisconsin in 2015 was 5.78 million.



A-1-8

There is a fascinating question in the "Review of Basic Inequalities" module (on Page 225) about a firm that manufactures desserts and small snacks. They experience a windfall when an airline decides to adopt their almond cookies for sale aboard their flights. On that page, we deal with the firm's short-term abilities to fulfill its orders. At this time, however, we'd like to address how to measure the cost of an expensive ingredient, almond extract, in their cost calculations. The facts are as follows: first, one package of almond cookies requires 2 fluid ounces of almond extract; second, their long-term contract with a new supplier will provide for almond extract at \$ 80 per gallon; third, a quick internet search reveals that there are 128 fluid ounces in one gallon. Your CEO wants to know exactly how much, per package, is the almond extract costing the company?

$$\begin{aligned}
\frac{2 \text{ fl oz}}{1 \text{ package}} \times \frac{1 \text{ gallon}}{128 \text{ fl oz}} \times \frac{80 \text{ dollars}}{1 \text{ gallon}} &= \\
\frac{2 \cancel{\text{ fl oz}}}{1 \text{ package}} \times \frac{1 \cancel{\text{ gallon}}}{128 \cancel{\text{ fl oz}}} \times \frac{80 \text{ dollars}}{1 \cancel{\text{ gallon}}} &= \\
\frac{2 \times 1 \times 80 \text{ dollars}}{1 \times 128 \times 1 \text{ package}} &= \\
\frac{160 \text{ dollars}}{128 \text{ package}} &= 1.25 \text{ dollars/package}
\end{aligned}$$



A-1-9

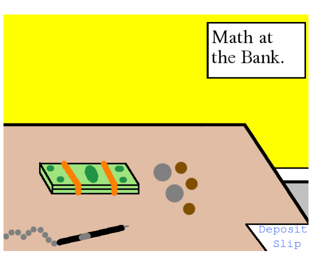
Let's suppose that you work for a company that equips factories with machinery. Your particular office is concerned with the hydraulics and pumps. A new catalog lists the pumping rate of some pumps in gallons per hour. However, your boss is much more used to the rate being measured in pounds of water per minute. One particular pump is rated at 217 gallons per hour.

A quick internet search reveals the following two facts:

- There are $3.78541\cdots$ liters in a gallon.
- A liter of water weighs a kilogram, by definition.
- A kilogram is $2.20462\cdots$ lbs.

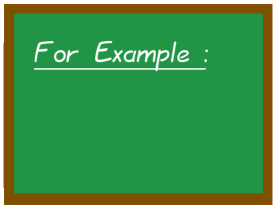
What is the pumping rate of this pump, in pounds per minute?

[Answer: $30.1824\cdots$ pounds per minute.] In the next box, we're going to talk about excessive precision and this answer.



In business or commerce, we should note the answer to the previous problem as 30.2 pounds per minute, because we only have three digits of precision from the 217 gallons per hour. Giving an excessive number of digits of precision might make you look odd.

However, in this textbook, we are using six digits of precision for all problems unless otherwise indicated. This way, you don't have to stress out about how many digits to show and how many to cut.



A-1-10

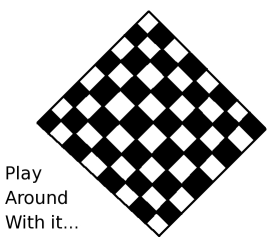
As a professor, I want to discourage my students from skipping class. To help make my point, I will compute how much money they are throwing away when they skip a class. One possible approach is to consider tuition. In the 2014–2015 academic year at UW Stout, each credit costs \$ 233.81 for a Wisconsin resident. My courses Math-123: *Finite & Financial Mathematics* and Math-154: *Calculus II* both meet four times a week and are 4-credit classes. The challenge before us is to calculate the cost of one missed class. We need another fact: there are 14 weeks per semester.
The mathematics is straight forward, and given in the next box.

Continuing with the previous box,

$$\begin{aligned}
\frac{233.81 \text{ dollars}}{\text{credit semester}} \times \frac{1 \text{ semester}}{14 \text{ weeks}} \times \frac{1 \text{ week}}{4 \text{ classes}} \times 4 \text{ credits} &= \\
\frac{233.81 \text{ dollars}}{\cancel{\text{credit semester}}} \times \frac{\cancel{1 \text{ semester}}}{\cancel{14 \text{ weeks}}} \times \frac{\cancel{1 \text{ week}}}{4 \text{ classes}} \times \cancel{4 \text{ credits}} &= \\
\frac{233.81 \times 1 \times 1 \times 4 \text{ dollars}}{1 \times 14 \times 4 \text{ class}} &= 16.7007 \text{ dollars/class} \approx \$\ 16.70 \text{ per class}
\end{aligned}$$

Therefore, we can conclude that someone who misses a class is throwing away \$ 16.70. By the way, to ensure that the previous box and the next box are accurate, I checked my old course calendars. Indeed, both classes meet 56 times during the semester, and $(14)(4) = 56$, so things look good.

However, the approach in this box is flawed, because it fails to take into account fees.

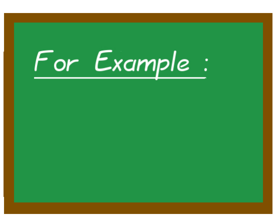


A-1-11

If you include tuition and fees together, you get a better picture. After all, the fees do have to be paid. Tuition is not the only cost of going to college!

- Including both tuition and fees, a typical student paid \$ 3609.96 for 12 credits in the 2014–2015 academic year at UW Stout.
- As noted before, each course is 4 credits and meets 4 times per week, and there are 14 weeks of classes each semester.

With all this in mind, how much does missing one class cost?
[Answer: $21.4878\cdots$ dollars/class ≈ \$ 21.49 / class.]



A-1-12

Let's suppose that Alice, Bob and Charlie are working together on this module. They've decided to practice together, and they just completed the previous box. Alice did this

$$\frac{3609.96 \text{ dollars}}{12 \text{ credits semester}} \times \frac{1 \text{ semester}}{14 \text{ weeks}} \times \frac{1 \text{ week}}{4 \text{ classes}} \times 4 \text{ credits}$$

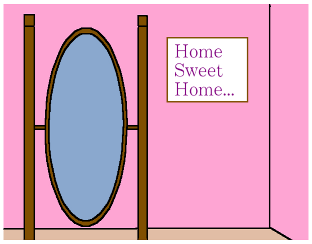
while Bob did this

$$\frac{1 \text{ semester}}{14 \text{ weeks}} \times \frac{1 \text{ week}}{4 \text{ classes}} \times \frac{3609.96 \text{ dollars}}{12 \text{ credits semester}} \times 4 \text{ credits}$$

and finally Charlie did this

$$\frac{1 \text{ week}}{4 \text{ classes}} \times \frac{1 \text{ semester}}{14 \text{ weeks}} \times \frac{3609.96 \text{ dollars}}{12 \text{ credits semester}} \times 4 \text{ credits}$$

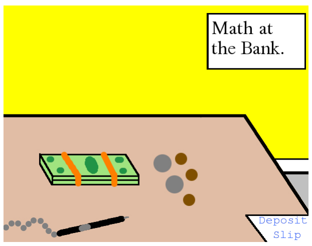
Should they be worried? They had different starting steps! They are nervous that they have gotten the problem wrong. What do you think? We'll continue in the next box.



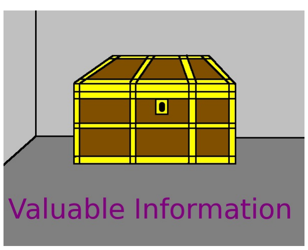
A Pause for Reflection. . .
We all know that the order of some numbers in a long multiplication is completely unimportant. For example,

$$(7)(4)(3) = (7)(3)(4) = (4)(3)(7) = (4)(7)(3) = (3)(7)(4) = (3)(4)(7)$$

Therefore, Alice, Bob, and Charlie, have written *precisely and exactly the same thing.* In a long multiplication, the order of the numbers being multiplied does not matter at all. They have no cause to be worried.



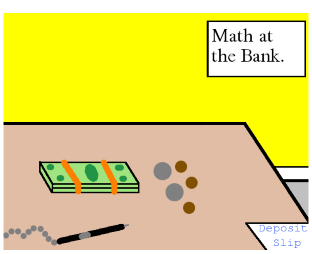
I'd like to take a brief moment now to talk to you about currencies. One of the worst things that can happen to a young person with business aspirations is if a job candidate, or worse, a new hire, sees the symbol € and doesn't know what it means. Not only would this be spectacularly damaging to that young person's career path, it would probably result in that company never again considering to hire anyone from that young person's university, ever.



The symbol € represents the currency called the Euro. In other words, just as the writing of "$ 15" indicates "fifteen dollars," the writing of "15 €" means "fifteen euros."

By the way, not all European Union (EU) member nations use the Euro. For example, Denmark, Sweden, and Poland are member nations of the EU, but do not use the Euro (as of the writing of this paragraph in January of 2017). Similarly, Kosovo, Montenegro, and Monaco—where I taught a course once in October of 2009—use the Euro, but are not member nations of the EU.

This information is subject to change. For example, the infamous "Brexit" vote occurred on June 23, 2016, after most of this module was written. During that vote, the citizens of the UK voted to leave the EU. That has not yet happened, and is estimated to take place slowly over time until March of 2019. Prior to the Brexit vote, the UK was the most famous example of an EU nation that does not use the Euro—they use the pound sterling as their currency. However, I cannot use them an example anymore, because they are leaving the EU now.



The nominal GDP of the EU and the USA are not equal, but are fairly close. For example, in 2015, according to The International Monetary Fund, the GDP of the EU was 16.220 trillion US dollars while the GDP of the USA was 17.947 trillion US dollars.

Those GDPs are remarkably close. More precisely, the GDP of the EU was only 9.62% less than the GDP of the USA. As you can see, an American business student not knowing what the euro is, would be comparably embarrassing to some European business student who had never heard of the US dollar.

One last interesting note is that the GDP of the entire world, from the same data set, was 73.171 trillion US dollars. That means that these two entities, the EU and the USA, alone comprise 46.69% of the world's GDP.

Below, I've listed the ten largest economies (in terms of nominal GDP) according to the data of The International Monetary Fund for 2015. This table is intended to give you a casual overview of the top ten nations. Of course, it goes without saying that you should not attempt to memorize the information in this box!

By the way, these Top 10 countries alone represent 66.83% of the world's GDP for 2015, and if you go to the Top 20 countries, then you get 80.49% of the world's GDP. That's an important remark for anyone interested in international business—the world's wealth is highly concentrated in a very small number of nations.

Rank	Country	GDP in US Dollars	Name of Currency	Currency Symbol
1.	The USA	$ 17,947,000,000,000	The US Dollar	$ or USD
2.	China	$ 10,983,000,000,000	The Renminbi or Yuan	¥ or RMB
3.	Japan	$ 4,123,000,000,000	The Yen	¥ or JPY
4.	Germany	$ 3,358,000,000,000	The Euro	€ or EUR
5.	The UK	$ 2,849,000,000,000	Pounds Sterling	£ or GBP
6.	France	$ 2,412,000,000,000	The Euro	€ or EUR
7.	India	$ 2,091,000,000,000	The Indian Rupee	INR
8.	Italy	$ 1,816,000,000,000	The Euro	€ or EUR
9.	Brazil	$ 1,773,000,000,000	The Brazilian Real	BRL
10.	Canada	$ 1,552,000,000,000	The Canadian Dollar	$ or CAD

I assume that you already know the symbol $, so the only three symbols that you need to learn are € for the Euro, ¥ for the Japanese Yen or Chinese Yuan, and £ for the British currency, pounds sterling. This should not be too hard.

In some ways, that list looks like a list of the big players in WWII (1939–1945), except with the insertion of India and Brazil, and the exclusion of Russia. Actually, India and Brazil both played significant roles in WWII, but our American textbooks de-emphasize that for political reasons. We'll see that Russia can be found in the next box.

I thought that you might be curious to see the next ten countries. Here are the 11th through 20th. As you can see, this is a more diverse list.

Rank	Country	GDP in US Dollars	Name of Currency	Currency Symbol
11.	South Korea	$ 1,377,000,000,000	The South Korean Won	KRW
12.	Russia	$ 1,325,000,000,000	The Russian Ruble	RUB
13.	Australia	$ 1,224,000,000,000	The Australian Dollar	AUD
14.	Spain	$ 1,200,000,000,000	The Euro	€ or EUR
15.	Mexico	$ 1,144,000,000,000	The Peso	MXN
16.	Indonesia	$ 937,000,000,000	The Indonesian Rupiah	IDR
17.	Netherlands	$ 738,000,000,000	The Euro	€ or EUR
18.	Turkey	$ 734,000,000,000	The Turkish Lira	TRY
19.	Switzerland	$ 665,000,000,000	The Swiss Franc	CHF
20.	Saudi Arabia	$ 653,000,000,000	The Saudi Riyal	SAR

Of course, there is no need to memorize this list!

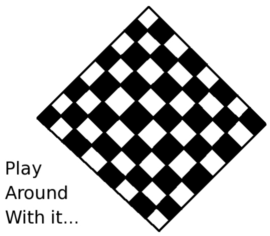
Play Around With it...

A-1-13

Jane is traveling in Brazil and is having a great time. She is considering moving there, and looks at some posted jobs in her area of specialization. One particular possibility looks interesting to her, and is listed as 18,500 Brazilian Reals per month. That sounds really high. She is curious to find out how much that is, in US dollars per year.

To help her, you look up the exchange rate online and see that 3.41700 Brazilian Reals is equal to one dollar. (By the way, this exchange rate was true on June 20th, 2016.)

How much is the equivalent salary in US dollars/year? [Answer: $ 64,969.27/year.]



A-1-14

Consider an Information Technology conference taking place in Brussels. Three attendees—each roughly mid-career professionals—have struck up a friendship. After a few too many drinks, they wish to compare the sizes of their salaries.

- Wages in the USA are often specified as dollars per hour. Charles says that he is paid \$ 36/hour. Remember, there are 40 working hours per week, and 52 weeks per year.
- Wages in France are often specified as Euros per month. Francois gets paid 6000 Euros per month.
- Wages in the UK are often specified as pounds sterling per annum. Percival says that he gets paid 50,000 pounds per annum. (Of course, "per annum" means "per year" just like the words "annually" or "anniversary.")

First, convert their salaries to dollars per year, to help these three gentlemen make their comparison. Then tell me who has the biggest salary, and who has the smallest. The answer will be given on Page 1172. Of course, you will need the exchange rates. On June 20th, 2016, the following rates were available:

1.45838 US dollars = 1 British Pound Sterling   1 US dollar = 0.881368 Euros

Often, when solving a problem like the one in the previous box, students will write \$ 69,120 per year for Charles's salary, instead of \$ 74,880 per year. Usually, the following work accompanies such an answer:

$$\frac{36 \text{ dollars}}{1 \text{ hour}} \times \frac{40 \text{ hours}}{1 \text{ week}} \times \frac{4 \text{ weeks}}{1 \text{ month}} \times \frac{12 \text{ months}}{1 \text{ year}} = \frac{36 \times 40 \times 4 \times 12}{1 \times 1 \times 1 \times 1} = 69,120 \neq 74,880$$

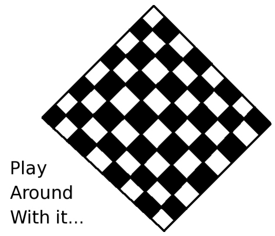
The reason this gives you the wrong answer is because the above work assumes there are 4 weeks per month, but that is completely wrong. It is not true that there are four weeks per month. We will explore this issue some more, in the next box.

Here are some simple ways to remember that it is not the case that there are four weeks in a month.

- If it were the case that there were 4 weeks per month, since there are 7 days per week, there would be $4 \times 7 = 28$ days per month. But of course, all the months except February have 30 days or 31 days, so clearly it is not the case that there are 4 weeks per month.
- If it were the case that there were 4 weeks per month, since there are 12 months per year, there would be $4 \times 12 = 48$ weeks per year. But of course, there are 52 weeks per year! So once again, clearly it is not the case that there are 4 weeks per month.
- If it were the case that there were 4 weeks per month, each wall calendar would have four rows of boxes with the dates in them. But, if you look at a wall calendar, you will see that there are five rows of boxes.

In summary, 4 weeks $\neq$ 1 month.

Just to be precise, there are $52\frac{1}{7}$ weeks in most years, and $52\frac{2}{7}$ weeks in leap years. For simplicity, most people just round this to 52 weeks per year, as I did in the middle bullet of the previous box.



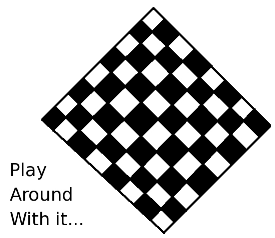
A-1-15

In 2009, I was teaching a week-long doctoral-level course, at the International University of Monaco. I was speaking with some employees of the hotel that I was staying at, remarking at the vastly higher standard of customer service in Monegasque hotels compared to American hotels. They replied that the minimum wage in Monaco is much higher than the minimum wage in the USA, and they felt that their workers are happier as a result, improving their emotional outlook, and reducing their overall grumpiness.

I decided to return to this problem in June of 2016. At that time, the minimum wage in Monaco was 1,715.94 euros per month, whereas in the USA, it is \$ 7.25 per hour.

- Knowing that $0.877666\cdots$ euros equals one dollar, and that there are 12 months in a year, what is the Monegasque minimum wage in dollars per year?
  [Answer: \$ 23,461.40 per year.]
- Knowing that there are 52 weeks in a year, and 40 hours in a work week for a standard minimum wage job, what is the US minimum wage in dollars per year?
  [Answer: \$ 15,080 per year.]

Well, it might be hard to know whether the minimum wage is the cause of better customer service. Nonetheless, it is spectacularly clear that the minimum wages in Monaco and in the USA are very different.

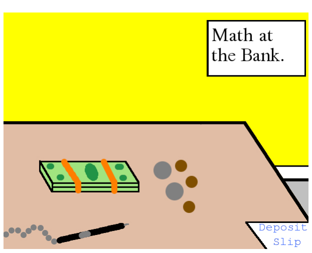


A-1-16

Suppose that Fred is traveling in Russia, to make a sales call for his company, and he sees that an extremely fine Beluga caviar costs 240,000 Russian Roubles per kilogram. He is curious about what that really means, so he'd like to convert it to dollars per pound. Surely that's how prices are marked in delis in the USA. Can you help him compute this?

You look up the exchange rate online and see that 64.47455 Russian Rubles is equal to one dollar (which was true on June 20th, 2016, by the way), and that $2.20462\cdots$ lbs is equal to one kilogram.

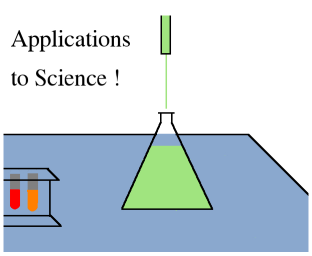
What is the equivalent price, in dollars per pound? [Answer: \$ 1688.45 / lb.]



That is a lot of money to throw away on a bunch of pickled fish eggs, but keep in mind that a typical serving would be around 50 g or 100 g, not 1000 g. Such a serving would cost between \$ 84.42 and \$ 168.84.

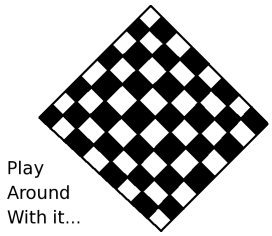
According to the article "Most Expensive Foods in the World," there are some caviars that cost 5000 US dollars per kilogram, which is about triple the cost of the caviar discussed in this problem.

`http://binscorner.com/pages/m/most-expensive-foods-in-the-world.html`



You'll notice that I always tell you how many feet are in a mile, or how many grams are in a pound. Often times, students ask me if they should memorize those quantities.

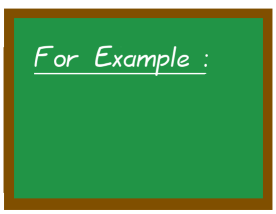
The answer is a decided and emphatic "no." When I was working at the Naval Surface Warfare Center, I would hang out with some of the physicists there, most of whom had PhDs. Many of them carried a business-card sized reference chart that had commonly needed unit conversions on them. Surely if a Naval Surface Warfare Center physicist does not memorize unit conversions, then you should not memorize them either.



\# A-1-17

At the Naval Surface Warfare Center in Potomac, Maryland, there is an instrument to measure how various hulls and shapes respond to contact with the water. While this is normally used for designing ships, torpedoes, and submarines, it can also be hired out for other purposes, such as to help design yachts. In fact, in the Summer of 2003, when I was working there, the instrument was used to judge the Human-Powered Submarine Contest. The instrument has a basin of water that is 21 feet wide, 10 feet deep, and 1048 feet long. (That is not a typo—the instrument is over one thousand feet long.)

To explain this to a football player, determine how long the basin is, measured in "football fields." Note that a football field is 120 yards long (including both endzones) and that 3 feet make a yard. [Answer: $2.91\overline{1}$ football fields.]



\# A-1-18

Let's continue with the previous box. How many gallons are in the basin, if it is full? Note that a gallon is $0.133680\cdots$ cubic feet.

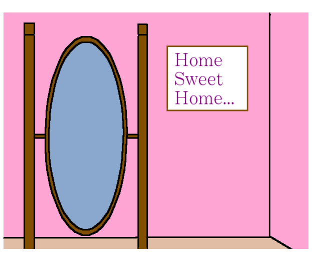
First, we have to find the volume of the basin.

$$(21 \text{ ft})(10 \text{ ft})(1048 \text{ ft}) = 220,080 \text{ ft}^3$$

Next, we should convert 220,080 ft^3 to gallons. That's very easy for us now.

$$220,800 \cancel{\text{ft}^3} \frac{1 \text{ gallon}}{0.133680 \cancel{\text{ft}^3}} = 1,646,319 \text{ gallons}$$

Indeed, it is rather a lot of water.



A Pause for Reflection...

Let's imagine that Alice and Bob are working on some shipping issues. They are planning to load a container for shipment on a container ship. The construction company they are working with is supplying some materials in a crate that is a cube, 2 yards of height, width, and length. However, their shipping plan is organized in terms of feet, square feet, and cubic feet.

Now there is a mild dispute. Bob thinks that since a yard is three feet, that a cubic yard is three cubic feet. In contrast, Alice thinks that since $3^3 = 27$, that a cubic yard is 27 cubic feet.

Take a moment, and think about this. Who is correct? Really, take a moment, and consider this point.

Now I'm going to reveal the answer to the question that I posed to you in the previous box. Therefore, stop reading now, unless you have taken some time to consider the thoughts of the previous box.

Our crate is 2 yards by 2 yards by 2 yards. Surely, because a yard is three feet, two yards is six feet. Therefore, our crate is 6 feet by 6 feet by 6 feet. Clearly, its volume is

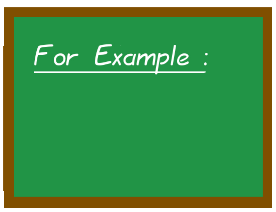
$$6 \text{ ft} \times 6 \text{ ft} \times 6 \text{ ft} = 216 \text{ ft}^3$$

As you can see, 8 cubic yards is equal to 216 cubic feet. Therefore, it is fantastically obvious that Bob is entirely wrong and Alice is entirely right.

Another way to look at the previous box is to realize that a cubic yard is 1 yard by 1 yard by 1 yard. Therefore, it is 3 feet by 3 feet by 3 feet. Thus the area of a cubic yard is

$$3 \text{ ft} \times 3 \text{ ft} \times 3 \text{ ft} = 27 \text{ ft}^3$$

That's another way to realize that we have to cube the 3:1 relationship. Next, I'm going to show you an even easier technique for dealing with units like cubic centimeters and cubic feet.



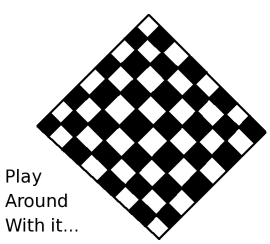
A-1-19

Suppose I am thinking of buying a portable printer to go with my laptop. It is made in Taiwan, and the website I'm looking at says that the volume of the printer is 4000 cc. (The abbreviation cc means "cubic centimeters," which can also be written as cm^3.) I would like to know what that means in terms of cubic inches.

The only thing that I need to do is rewrite cm^3 as "cm times cm times cm." Likewise, cubic inches or in^3 will be rewritten as "in times in times in." This is just like when I write $8^3 = 8(8)8$ or $7^3 = 7(7)7$. Once I make this adjustment, everything else just works out nicely.

$$\begin{aligned}
4000 \text{ cm}^3 = 4000 \text{ cm cm cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} &= \\
4000 \cancel{\text{ cm cm cm}} \times \frac{1 \text{ in}}{2.54 \cancel{\text{ cm}}} \times \frac{1 \text{ in}}{2.54 \cancel{\text{ cm}}} \times \frac{1 \text{ in}}{2.54 \cancel{\text{ cm}}} &= \\
\frac{4000 \times 1 \times 1 \times 1 \text{ in in in}}{2.54 \times 2.54 \times 2.54} &= 244.094\cdots \ \text{in}^3
\end{aligned}$$

Well, now I know that the volume of the printer is 244.094 cubic inches. To see if you've got this down, and to better understand what that means, let's convert this into cubic feet in the next box.

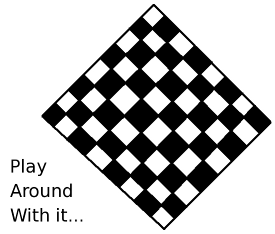


A-1-20

Let's start with the printer (from the previous box) being 244.094 cubic inches. What is that in cubic feet? Recall, there are 12 inches in a foot.

[Answer: $0.141258\cdots$ ft^3.]

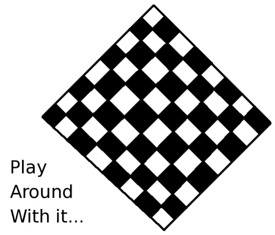
Since we're talking about centimeters now, I should remind you that 100 cm is one meter. I presume that most of my readers already know this. The full list of metric prefixes is given on Page 1170 for your reference. However, there is no need to memorize those. The three most common are "kilo-," "centi-," and "milli-," which you surely already know. (The prefix "milli-" means "one thousandth.")



A-1-21

Let's suppose you get a job with a petroleum company. They are setting up an office in Kuwait. Because it is a new office, the budget is very tight, and the company needs you to choose a small apartment, to keep expenses down. One particular apartment is listed as having 21 square meters of floor plan. Of course, you are used to reading advertisements for American apartments or dorm rooms, which are listed in square feet. A quick internet search reveals that 1 meter is 3.28084 feet.

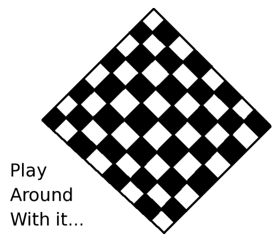
What is that area in square feet? [Answer: 226.042 square feet.]



A-1-22

Continuing with the previous box, it is common to have a high-performance air filter, because a lot of sand and dust blows in from the deserts nearby. Before departure, you found one in your home town, rated for up to 2500 cubic feet. Assume that the ceilings are 2.5 m above the floor, which is typical.

- What is the volume of the apartment in cubic meters? [Answer: 52.5 cubic meters.]
- What is the volume of the apartment in cubic feet? [Answer: 1854.02 cubic feet.]
- Is your air filter sufficient? [Answer: yes.]



A-1-23

Suppose you are working for an environmental clean up company, that takes care of old factories. In some documents, you read that the density of lead is 11.34 g/cc. (Note that g/cc means "grams per cubic centimeter.") To understand what that means, it is useful to convert that into pounds per cubic inch. Note that a kilogram is $2.20462\cdots$ pounds, and an inch is exactly 2.54 centimeters.

What is the density of lead in pounds per cubic inch? [Answer: 0.409683 lbs/in^3.]

By now, you know how to solve the problem in the previous checkerboard box. The correct setup is as follows:

$$\frac{11.34\text{ g}}{1\text{ cm}^3}\times\frac{1\text{ kg}}{1000\text{ g}}\times\frac{2.20462\text{ lb}}{1\text{ kg}}\times\frac{2.54\text{ cm}}{1\text{ in}}\times\frac{2.54\text{ cm}}{1\text{ in}}\times\frac{2.54\text{ cm}}{1\text{ in}}=\frac{0.409683\text{ lbs}}{\text{in}^3}$$

However, let's consider the possibility that I did the work of the previous problem this way:

$$\frac{1\text{ cm}^3}{11.34\text{ g}}\times\frac{1000\text{ g}}{1\text{ kg}}\times\frac{1\text{ kg}}{2.20462\text{ lb}}\times\frac{1\text{ in}}{2.54\text{ cm}}\times\frac{1\text{ in}}{2.54\text{ cm}}\times\frac{1\text{ in}}{2.54\text{ cm}}=\frac{2.44091\cdots\text{ in}^3}{\text{lb}}$$

It would only be natural to be dismayed. After all, I have done a lot of work, and a lot of writing. While I was asked to find pounds per cubic inch, I instead have found cubic inches per pound, which is wrong. Do I have to start the problem over?

It turns out that you don't have to start the problem over! My units are upside down. The mathematical way to describe this is that I have found the reciprocal units of what I actually wanted. To salvage this, I simply take the reciprocal on my calculator.

$$\frac{1}{2.44091\cdots}=0.409683\cdots$$

and I now have the correct answer! This trick saves me from having to do the problem twice, in the event that I make the error described above.

Play Around With it...

Suppose a friend of yours is shopping for some fabric, to make a Sari (an Indian dress for formal occasions). Visiting NYC, she sees some silk fabric with a very fancy floral pattern, suitable for making a nice Sari. It is selling for $ 6.25 per square foot. She wants to call her mother in India, to see if this is a good price or not. Convert this price into rupees per square meter. One dollar was 68.0422 rupees on January 12th, 2017. Recall, there are 12 inches per foot and 2.54 cm per inch.

[Answer: 4577.50 rupees per square meter.]

A-1-24

Play Around With it...

Suppose the mother of a friend of yours is vacationing in China, and sees some very fancy damask wallpaper on sale. She suspects the price is much lower than in the USA. The wallpaper goes for 168 Chinese Yuan per square meter. Convert this to dollars per square foot so that she can compare it to what she's used to. On January 12th, 2017, one dollar was worth 6.9018 Chinese yuan. There are 12 inches in a foot, and 1 inch is exactly 2.54 cm.

[Answer: $2.26139\cdots$ dollars per square foot.]

A-1-25

Play Around With it...

For this problem, perhaps you are setting up a software lab in Brazil. The computer room has to be air-conditioned, of course, to protect the servers, and that means that it needs good insulation. You contact a friend of yours in the USA, who tells you that quality insulation often costs $ 4 per cubic foot. To guide your shopping, tell me how much that is in Brazilian Reals per cubic meter. Note that $1 was worth 3.18780 Brazilian Reals on January 12th, 2017. There are 12 inches in a foot, and 1 inch is exactly 2.54 cm.

[Answer: $450.304\cdots$ Brazilian Reals per cubic meter.]

A-1-26

Play Around With it...

Let's imagine that you're showing a Japanese client the sights in New York City. You are visiting a famous auction house, to witness an art auction. Today, there is a contest offering a cube of gold, 2 inches on each side, to whomever will win a trivia contest about art history. You have to get all the questions correct to win the cube, which is a hint that the cube is probably worth a great deal of money.

You both are curious about how much this cube of gold is worth, and to figure that out, you need the price of gold and the density of gold.

We'll continue in the next box.

A-1-27

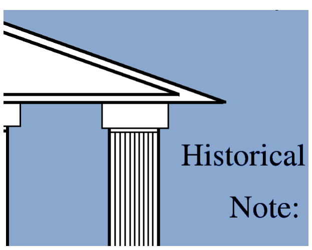
Continuing with the previous box, you look up those needed facts on your smartphone.

- The density of gold is 19.32 grams per cubic centimeter.
- A kilogram is $2.20462\cdots$ pounds.
- The price of gold (on June 20th, 2016), was \$ 41,860.24 per kg, in US Dollars.
- One dollar was worth 104.695 ¥ (on June 20th, 2016).

With all that in mind, please answer the following questions:

Hint: I leave it up to you to determine in what order the following questions should be answered. It is actually easier to answer them in a different order than the order in which I have written them. Just think of this as an extra challenge.

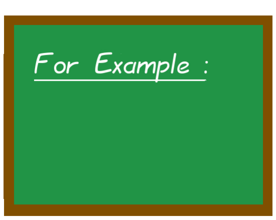
- What is the volume of this cube, in cubic inches? [Answer: $2(2)2 = 8$ cubic inches.]
- What is the value of the cube, in yen? [Answer: $11,100,075.\cdots$ ¥.]
- What is the value of the cube, in dollars? [Answer: \$ $106,022.\cdots$.]
- What is the weight of the cube, in pounds? [Answer: $5.58382\cdots$ lb.]
- What is the mass of the cube, in kilograms? [Answer: $2.53278\cdots$ kg.]



I've always thought it was funny that pounds are abbreviated "lbs," because there is no "L" in pound, nor is there a "B" in pound. What a crazy abbreviation!

The abbreviation "lb" is from the Latin *libra*, which means balances or scales. The Roman unit of weight was a *libra pondo*. When someone would weigh items at a Roman market, of course, you'd have to use a scale.

Having faulty scales, of course, would allow a merchant to cheat customers or fellow merchants. Therefore, there was an official who would use a test weight, whose exact weight was known, to check the scales of all the merchants periodically. However, there is a flaw in this method—the test weight could be modified or trimmed just a bit, cutting off small amounts. To prevent this, the test weight was often a fancy carving, such as the bust of a Roman god or the Roman Emperor. Cutting off small chunks from that would immediately make the modification obvious, ruining the carving. Several such weights are extant.



A-1-28

Let's suppose that you are working in a factory, and you have to purchase some industrial strength insulation. An American supplier has a suitable product which weighs 159 pounds per cubic foot. An Italian supplier also has a suitable product at 2.51 grams per cubic centimeter. If you have two slabs of identical dimensions, one of each material, which one will be heavier: the Italian? or the American? (A more proper way of saying this is to ask which product is denser.)

One way to solve this problem is to convert the Italian product into the American units of pounds per cubic foot. We'll do that in the next box.

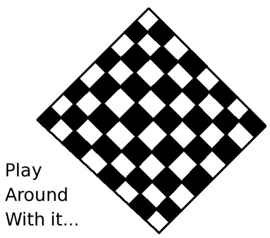
Here's my solution to the problem of the previous box.

$$\frac{2.51\text{ g}}{1\text{ cm cm cm}} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{1\text{ kg}}{1000\text{ g}} \times \frac{2.20462\text{ lb}}{1\text{ kg}} =$$

$$\frac{2.51\,\cancel{\text{g}}}{1\,\cancel{\text{cm}}\,\cancel{\text{cm}}\,\cancel{\text{cm}}} \times \frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}} \times \frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}} \times \frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}} \times \frac{12\,\cancel{\text{in}}}{1\text{ ft}} \times \frac{12\,\cancel{\text{in}}}{1\text{ ft}} \times \frac{12\,\cancel{\text{in}}}{1\text{ ft}} \times \frac{1\,\cancel{\text{kg}}}{1000\,\cancel{\text{g}}} \times \frac{2.20462\text{ lb}}{1\,\cancel{\text{kg}}} =$$

$$\frac{2.51 \times 2.54 \times 2.54 \times 2.54 \times 12 \times 12 \times 12 \times 1 \times 2.20462\text{ lb}}{1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1000 \times 1\text{ ft ft ft}} = 156.693\cdots\text{lb/ft}^3$$

Clearly, the American insulation is denser.



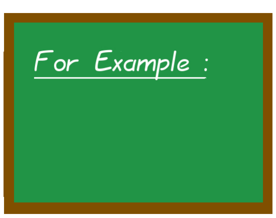
A-1-29

The alternative strategy would be to convert the American product's density from 159 pounds per cubic foot into grams per cubic centimeter.

Knowing that there are 12 inches in a foot, that an inch is exactly 2.54 cm, a kilogram is 1000 grams, and that a kilogram is $2.20462\cdots$ pounds, what is 159 pounds per cubic foot equal to, in grams per cubic centimeter? [Answer: $2.54693\cdots$ g/cc.]

As before, the American product is slightly denser, so two slabs of equal dimensions would have the Italian product lighter and the American product denser.

The following extremely challenging problem was suggested by my husband, Patrick Studdard.



A-1-30

Some European magazines measure the fuel efficiency of automobiles in liters per 100 km. This makes some sense, because if you're planning a 200 km or 300 km vacation, then you will know how much gasoline you will probably be using.

However, this can also be maddening, because in mpg, a higher number is very fuel efficient car, but a lower number is a less fuel efficient car. With liters per 100 km, the higher number is a less fuel efficient car, but the lower number is a more fuel efficient car.

Earlier, on Page 1153, we had a problem about a particular Toyota Prius, whose owner claimed 51 mpg. Let's contrast this vehicle with a particular Cadillac Escalade, getting 15 mpg. I will solve the problem for the Cadillac Escalade myself, and then you will solve the problem for the Toyota Prius yourself.

We need to know that there are 3.78541 liters to the gallon, and one mile is 1609.34 meters. The trick here is that we're going to make a "fake unit" called a "standard trip," which equals 100 km.

$$\frac{100\text{ km}}{1\text{ standard trip}} \times \frac{1000\text{ meters}}{1\text{ km}} \times \frac{1\text{ mile}}{1609.34\text{ meters}} \times \frac{1\text{ gallon}}{15\text{ miles}} \times \frac{3.78541\text{ liters}}{1\text{ gallon}} =$$

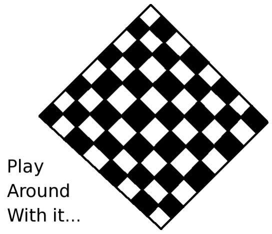
$$\frac{100\,\cancel{\text{km}}}{1\text{ standard trip}} \times \frac{1000\,\cancel{\text{meters}}}{1\,\cancel{\text{km}}} \times \frac{1\,\cancel{\text{mile}}}{1609.34\,\cancel{\text{meters}}} \times \frac{1\,\cancel{\text{gallon}}}{15\,\cancel{\text{miles}}} \times \frac{3.78541\text{ liters}}{1\,\cancel{\text{gallon}}} =$$

$$\frac{100 \times 1000 \times 1 \times 1 \times 3.78541\text{ L}}{1 \times 1 \times 1609.34 \times 15 \times 1\text{ standard trip}} =$$

$$15.6810\cdots\text{ L/standard trip}$$

Finally, because a "standard trip" is 100 km, we know that

$$15.6810\cdots\text{ L per standard trip} = 15.6810\cdots\text{ L per 100 km}$$

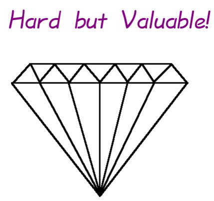


A-1-31

I'd like to challenge you now. I'd like you to reproduce the work of the previous box, but without looking at this module. Take an entirely blank piece of paper. Write the following facts, and then solve the problem using only what you have written on the blank page, without looking at this module.

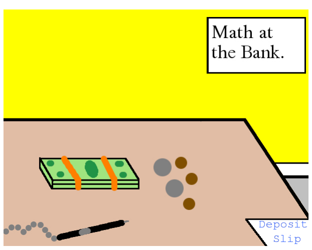
- There are 3.78541 liters to the gallon.
- One mile is 1.60934 kilometers.
- This particular Toyota Prius gets 51 mpg.
- We want to know how many liters per hundred kilometers.

The final answer will be given on Page 1172.

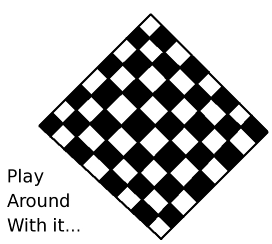


The primary content of this module has now been delivered. In the remaining pages, we will discuss a long problem about submarines. Then I will give a few paragraphs of description of the infamous incident where an error of unit conversions caused not one but two (!) of NASA's missions to Mars to be destroyed immediately upon arrival at Mars, without performing any of their scientific missions. Finally, I give some information about the metric system and its history.

This additional content is not essential, but might be of interest to readers who are thinking of a high-tech career path.



One of the most lucrative industries to work with is the US Defense industry. Because military hardware has to perform complex tasks under an array of challenging conditions, it is often very complicated. Because it is complicated, many hundreds of hours of engineering time are involved, which makes military hardware extremely expensive. In turn, that means that even with modest profit margins, the potential profits are in the hundreds of millions or the billions of dollars. Let's just spend two short boxes looking at a problem about submarines, to give you a taste of defense-related matters.



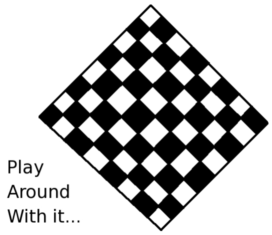
A-1-32

Submarines are very difficult to design because they have to be able to withstand extraordinary pressures. The performance characteristics of the submarines currently in use by the US Navy are (unsurprisingly) classified. I have found three sources for the old-fashioned *Los Angeles* class fast attack nuclear-powered submarines, which are being slowly retired from the fleet at this time. The three sources cite the maximum diving depth as 650 ft, 950 ft, and 1475 ft. Let's go with the middle estimate, namely, 950 ft.

At that depth, the water pressure is a surprising 28.7664 atmospheres. (That means that the pressure is 28.7664 times more than at sea level, or 29.7664 times more if you include atmospheric pressure as well as water pressure.) Knowing that one atmosphere is 14.6959 pounds per square inch (psi), compute the following:

- What is the water pressure at that depth, in psi? [Answer: 422.748 psi.]
- What is that water pressure, in pounds per square foot?
  [Answer: 60,875.7 pounds per square foot.]
- What is that water pressure, in tons per square foot? Recall that 2000 pounds is one ton. [Answer: 30.4378 tons per square foot.]

Typically the submarine will have a crew of 125–135, so a lot of lives are depending on the quality of the design.

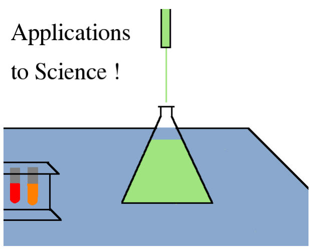


A-1-33

Looking again at the previous box, let's compute the answers to the same questions at a depth of 1475 feet. The water pressure at that depth is 44.6637 atmospheres. Knowing that one atmosphere is 14.6959 pounds per square inch (psi), compute the following:

- What is the water pressure at that depth, in psi? [Answer: 656.373 psi.]
- What is that water pressure, in pounds per square foot? [Answer: 94,517.7 pounds per square foot.]
- What is that water pressure, in tons per square foot? Recall that 2000 pounds is one ton. [Answer: 47.2588 tons per square foot.]

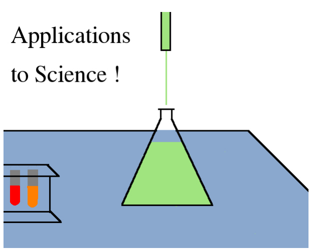
How important is it, exactly, to understand conversions between the metric system and the US customary units? If you want to do business with people, of course it is in your best interest to present them with information in such a way that they can understand it. However, I have a stronger argument: I will show you how expensive unit conversion mistakes can become.



The Mars Climate Orbiter, also called the Mars Surveyor '98 Orbiter, was an unmanned mission to Mars to study the atmosphere and climate of that planet. A lot of the science on board that spacecraft was related to climate change, finding water on Mars (important for any future manned missions or colonizations), and detecting whether there ever was life on Mars. In other words, it was a good scientific mission, and furthermore, being less than one billion dollars, it was unusually cheap by NASA standards.

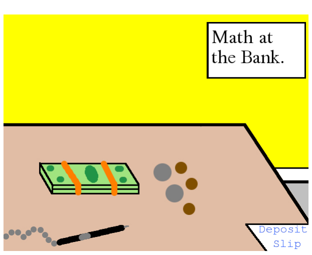
Something went horribly wrong on that mission, and it was all because of a confusion between US customary units and the metric system.

I will describe the debacle in the next box.



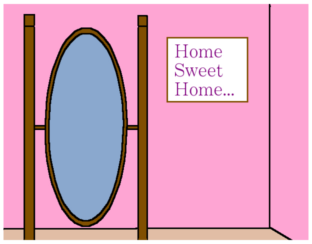
As it turns out, the ground-based computer software was using pounds as a unit in some calculation, and it transmitted data in pound-seconds to the spacecraft. The spacecraft's software was expecting the metric unit (Newton-seconds), and thus it had a completely different interpretation of the number which had been sent. As a result, when the orbiter began to maneuver itself to enter into orbit around Mars on September 23, 1999, it was at a very low altitude (due to the incorrect units) and ended up disintegrating in the Martian atmosphere. Of course, since this had happened very soon upon arrival at Mars, the spacecraft was destroyed before almost all the experiments could even start.

The total price of the mission was \$ 655,200,000, launching in December of 1998. Using what you will learn when you study inflation (starting on Page 455), you'll be able to calculate that this is equivalent to \$ 933,813,906 in the money of July of 2013. (Actually, it is hard to imagine inflation data accurate to six significant figures, so we should say 933.8 million dollars.) In the next box we will quantify this waste more precisely.



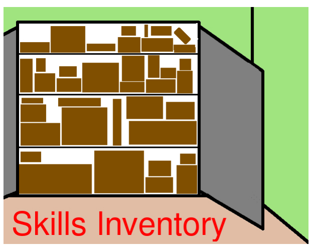
The US Population in January of 1999 was approximately 271,645,214, and the civilian labor force in December of 1998 was about 132,602,000 employed people. Thus the losses from the Mars Climate Orbiter mission come out to be approximately \$ 2.41 per person or \$ 4.94 per employed worker, in the money of December 1998.

The equivalent sums of money in July of 2013 would be \$ 3.43 per person or \$ 7.04 per employed worker. (You'll learn how to adjust for inflation yourself, on Page 455.)



A Pause for Reflection...
I think if I were to take \$ 7.04 from the wallet or pockets of some employed worker, without their consent and with virtually no outcome of any kind, they'd be rather mad at me. If I were to take \$ 7.04 from the wallets of thirty employed workers, without their consent or any outcome, I'd probably have to go to jail. So imagine how horrible it is to take \$ 7.04 from the pockets of almost 133 million employed workers.

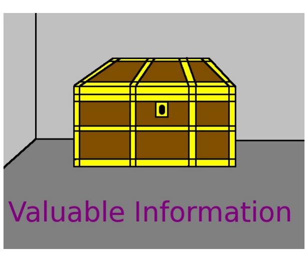
In summary, getting the units wrong in a math problem can be devastating. Even though converting units is a bit complicated (even NASA got it wrong, just this one time), you really should try to get it right.



You have now completed this module. Thank you for reading. Here's a summary of what you have been shown.

- How to convert between units, such as American units and metric units.
- How to include foreign currencies in such conversions.
- How to further apply that knowledge to areas, to pressures, to volumes, and to densities.
- Three different tricks for remembering that 1 month $\neq$ 4 weeks.
- You've also gotten some exposure to what units are used in which situations.

As promised, the next few boxes contain the full list of prefixes used in the metric system.

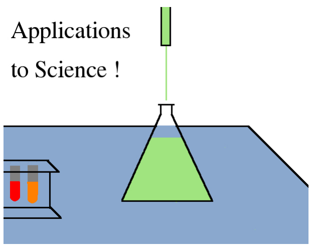


Here is a table of the metric prefixes. Some of them are common, but the ones with a ⋆ are not very common at all. If you know about electronics, you might know deci- from decibel, giga- from gigabyte, or nano- from nanosecond. These units can be applied to all sorts of things, including slang. For example: "megabucks" for millions of dollars, or "Agnes makes 85k," as we mentioned on Page 1153 of this module.

Terameter/Teragram/Teraliters	Tm/Tg/TL	10^{12}
Gigameter/Gigagram/Gigaliters	Gm/Gg/GL	10^9
Megameter/Megagram/Megaliters	Mm/Mg/ML	10^6
Kilometer/Kilogram/Kiloliters	km/kg/kL	10^3
⋆ Hectometer/Hectogram/Hectoliters	hm/hg/hL	10^2
⋆ Dekameter/Dekagram/Dekaliters	dam/dag/daL	10^1
Meter/Gram/Liters	m/g/L	$10^0 = 1$
Decimeter/Decigram/Deciliter	dm/dg/dL	10^{-1}
Centimeter/Centigram/Centiliter	cm/cg/cL	10^{-2}
Millimeter/Milligram/Milliliter	mm/mg/mL	10^{-3}
Micrometer/Microgram/Microliter	μm/μg/μL	10^{-6}
Nanometer/Nanogram/Nanoliter	nm/ng/nL	10^{-9}
⋆ Picometer/Picometer/Picoliter	pm/pg/pL	10^{-12}
⋆ Femtometer/Femtogram/Femtoliter	fm/fg/fL	10^{-15}

Next, I have some important notes in the next box.

- The symbol for micro- is the Greek letter μ, which plays the role of a lowercase "m" in their alphabet.
- There are two notable naming exceptions: a megagram is called "a metric ton", and a micrometer is called "a micron." These are actually important, because they are used often, not only in science but in industry. You'll hear "metric tons" all the time when discussing shipping, and lots of high tech industries will use the word micron.
- For whatever reason, it is very common to see smaller units of time (e.g. millisecond, microsecond, nanosecond) but rare to see large units of time. I've never seen kilosecond, megasecond, or gigasecond, except in theoretical astronomy.



There are actually some new prefixes above and below those we listed. Yet realistically, they are not very well known and are almost never used. All of them were created after 1960, as compared to the metric system itself, which is from 1795.

- The larger ones are "petameter" for 10^{15}, "exameter" for 10^{18}, "zettameter" for 10^{21}, and "yottameter" for 10^{24}.
- The smaller ones are "attometer" for 10^{-18}, "zeptometer" for 10^{-21}, and "yoctometer" for 10^{-24}.

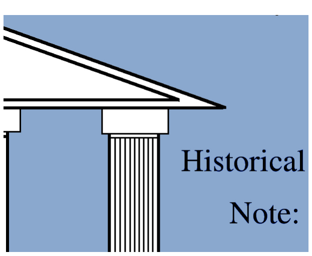
Unless you study nanomaterials, nuclear physics, or quantum mechanics, you will probably never see these again. You might see them in science fiction.

Technically, a kilobyte is 1024 bytes, not 1000, and so a megabyte is 1024^2 bytes, not 1000^2 bytes. Likewise a gigabyte is 1024^3 and not 1000^3 bytes. This isn't too important but at the terabyte level 1024^4 is about 10% more then 1000^4. Actually, most computer scientists don't know the facts in this box, so you should not fret over it. Someday soon we might hear about petabytes, which are 1024 terabytes or 1024^5 bytes, not 1000^5 bytes.

You might be wondering, why use 1024 and not 1000? Because of the following

$$1024 = 2^{10} = \text{binary}(10000000000)$$

and because computers use binary numbers.



Why do we have a metric system? Many students are taught that in the medieval period, units were based off the king's body parts (the length of his foot and the length of his arm, and so forth). Every time there was a new monarch, all the units changed. Clearly, this was impractical, and when the French Revolution beheaded Louis XVI (1754–1793), the revolutionary scholars created a new idealized system based on the number 10. This is obviously false for several reasons.

First of all, the idea of the units changing each time a new king came to power is beyond absurd, as many monarchs had remarkably short reigns, falling either to disease, war, or the occasional assassination. No thinking person could ever imagine for an instant that every scale, ruler, and measuring cup in an entire kingdom would be realigned each time this happened. Second, the commission to create a system of units was chartered by Louis XVI himself, and included major scholars such as Antoine-Laurent de Lavoisier (1743–1794), who discovered oxygen and hydrogen, but who was most definitely affiliated with the monarchy. In fact, Lavoisier was guillotined for his involvement in "Le Ferme Générale", the 18th century French equivalent of the US Internal Revenue Service (the IRS).

You may see these terms used in older documents, but they are not used today.

- At one time, hybrid prefixes were used. An example is "millicron" for nanometer, which I imagine might come from "milli-micron."
- In older electronics books, I have seen "micromicrofarad" for picofarad.
- Femtometers are often called fermimeters, after Enrico Fermi (1901–1954). A proton or neutron is about 1.6-1.7 fm, and Enrico Fermi was among those who worked on early nuclear physics. In particular, protons, neutrons, and their similar particles are called "fermions" in his honor.
- In the 19th century, it was common to use myria- for 10,000, thus a myriameter was 10 km or 6.21 miles, and a myriagram was 10 kg, around 22.03 lbs.

This is the solution to the problem, given on Page 1160, about the three IT techs who want to compare the sizes of their salaries. For Percival, we have

$$\frac{50,000 \text{ pounds}}{1 \text{ year}} \times \frac{1.45838 \text{ dollars}}{1 \text{ pound}} =$$
$$\frac{50,000 \cancel{\text{pounds}}}{1 \text{ year}} \times \frac{1.45838 \text{ dollars}}{1 \cancel{\text{pound}}} = \frac{50,000 \times 1.45838}{1 \times 1} = 72,919.0 \text{ dollars/year}$$

For Francois, we have

$$\frac{6000 \text{ Euros}}{1 \text{ month}} \times \frac{1 \text{ dollar}}{0.881368 \text{ Euros}} \times \frac{12 \text{ months}}{1 \text{ year}} =$$
$$\frac{6000 \cancel{\text{Euros}}}{1 \cancel{\text{month}}} \times \frac{1 \text{ dollar}}{0.881368 \cancel{\text{Euros}}} \times \frac{12 \cancel{\text{months}}}{1 \text{ year}} =$$
$$\frac{6000 \times 1 \times 12}{1 \times 0.881368 \times 1} = 81,691.1 \cdots \text{ dollars/year}$$

For Charles, we have

$$\frac{36 \text{ dollars}}{1 \text{ hour}} \times \frac{40 \text{ hours}}{1 \text{ week}} \times \frac{52 \text{ weeks}}{1 \text{ year}} =$$
$$\frac{36 \text{ dollars}}{1 \cancel{\text{hour}}} \times \frac{40 \cancel{\text{hours}}}{1 \cancel{\text{week}}} \times \frac{52 \cancel{\text{weeks}}}{1 \text{ year}} = \frac{(36)(40)(52) \text{ dollars}}{(1)(1)(1) \text{ year}} = 74,880 \text{ dollars/year}$$

As you can see Francois has the biggest salary, whereas Percival has the smallest salary.

Here is the answer to the challenging question from Page 1168, where you were asked to convert the mileage of a Toyota Prius, from 51 mpg, into the units "L per 100 km."

[Answer: $4.61205 \cdots$ L per 100 km.]

However, note that this is extremely shocking. Of course, a higher number in mpg represents "better" mileage. By better, we mean that the engine of the Prius will go further than the Escalade on the same amount of fuel, or use less fuel than the Escalade to go the same distance. In the European measure "L per 100 km," it is the reverse. A smaller number represents "better" mileage.

This can make comparisons very confusing.