The Preface—How to Use This Book

You might have some questions about the nature and purpose of this book. Why does this book organize the text in boxes? And why is it filled with cartoon-like drawings? Who is this book for and at what point in their education?

This textbook covers "Finite Mathematics," and "Financial Mathematics," two exciting and useful (but relatively easy) branches of mathematics. It is intended for freshmen or sophomores, particularly those majoring in the money-related fields. I define this broadly, to include finance, economics, marketing, accounting, management, human resources, international commerce, hotel & tourism management, business administration, and other majors.

The book is probably very useful for students in the social sciences as well. By the social sciences, I would include psychology, education, sociology, anthropology, archeology, geography, and especially political science.

One might imagine that this book *is not* intended for math majors, engineering students, nor physics students. However, I find that a lot of topics in probability and combinatorics were amputated from the high school curriculum long ago, due to "the common core." Yet, these topics are very useful and model circumstances that occur in those subjects. Therefore, even math majors, engineering students, and physics students can profit from reading those topics, covered in those late chapters.

The course covered is usually called *Finite Mathematics* at most universities in the USA or Canada, though some call it *Business Mathematics*. In the United Kingdom, the course is called *Quantitative Methods*. Essentially, it is all the mathematics that you need to know for a degree related to economics, finance, the managerial or social sciences, up to but not including calculus.

This book is very suitable for self-study; I hope home-schooled students will consider it also, as a capstone course after algebra, before beginning university study or before starting calculus.

Okay, so why the cartoons and the boxes? The format of this book is based on the idea of breaking the module up into tiny bite-sized easily-digested pieces. Certainly, this is bizarre. I've never seen a book written this way, so perhaps you have not either—but I've done this for several reasons:

- The book is designed primarily for electronic form, and the computer screen is much wider than it is tall, as compared to a piece of paper, which is much taller than it is wide.
- The idea is that you should read the boxes one at a time, and that you *must not read* the next box until you have understood the current box, except as noted below. If you must read the current box ten times to understand it, then indeed, read it ten times. The exceptions are certain types of boxes that are categorically tangential.

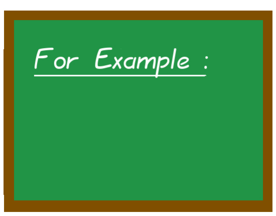
By breaking the text into these tiny pieces, each a bit further along than the previous, the idea is that you will ascend in mathematical knowledge smoothly and with minimal effort. It shall be like climbing the stairs, rather than scaling a cliff.

This book is not, however, an elevator. You will have to work a bit. I've gone through a lot of effort and typing to make this process easier for you, but it will not be "a breeze." There is no need to enjoy mathematics in order to learn it. In fact most students do not enjoy mathematics. Yet it remains true that learning mathematics requires some serious effort.

Nothing can replace the joy and feeling of accomplishment that comes after a long and difficult problem, checking the answer, and discovering that you are correct.

One of the things that I've done in writing this book is to try to identify, by going over old quizzes and tests, the set of mistakes that are most common among students in the freshmen math courses—including this course but also calculus. The contents of these boxes represent material and concepts that are often lost on students, leading to poor performance on exams or mathematical misconceptions.

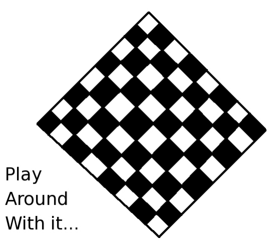
You can benefit from the mistakes of others by reading about them, and avoiding them. The boxes with this bomb in them are those warnings. These boxes are indicators that the concepts they contain are vitally important for mastery of the material.



0-0-1

The primary form of instruction will be examples. Seeing how a mathematical tool is used is a great way to acquire the knowledge of how to use it. I have tried to make the examples as real-world oriented as possible, even to the point of studying books normally used in higher-level finance courses, economics courses, and science courses.

The examples are the heart and soul of the book.



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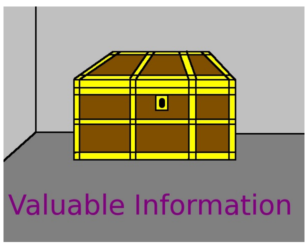
After each example will follow some practice problems for you. The question will be given, as well as the final answer. You must take a blank piece of paper and challenge yourself to try to get the correct answer. If you cannot, then try again and again. It is in this manner that we will pass from easy problems to harder ones—and there will be hard problems, I assure you.

By the way, do you see the numbers to the left? In this case, they are 0-0-2. This allows you to identify specific checkerboard boxes, so that you can discuss them with your instructor, help a classmate with one, or ask a question about one during class. For example, 3-2-5 means Chapter 3, Module 2, Problem # 5. By "problem," we include example problems (like the previous box) and practice problems (like this box, marked with a checkerboard) but no other boxes.

It is extremely important to get in the habit of checking your own work. For the strongest students, checking one's work in high school was optional. The problems were not long and complex enough to merit checking intermediate steps, except perhaps occasionally. Now, however, we will have a few extraordinarily long problems. It is crucial that you learn to check your work in intermediate steps, otherwise you will be unable to do the longer problems. These boxes will tell you how to check your work.

Boxes of this sort are meant to link our present discussion of some mathematical topic with a historical reference or notable person in the history of mathematics, finance or science. Many people enjoy discovering where a mathematical idea comes from or who invented it. Others do not care, and that's just fine—they can freely skip these boxes.

The book *Innumeracy* by John Allen Paulos (1945–present) was a huge influence upon the development of this work, and it opened the eyes of a nation to pitfalls of mathematical incompetence. Mainly, the reason that *Innumeracy* will be remembered centuries from now is that it gave us a new word. This word, *innumeracy*, is delightful. Its meaning will now be explained in the next box.



These boxes contain information that must be learned, at all costs. Only the most vital information is placed in these boxes, and you must be careful to learn what you find in them. Often they define core vocabulary or introduce key concepts.

An *illiterate* person is one who cannot read any written language. Such a person has a huge disadvantage in our present day, do they not? Truly, it is hard for us to grasp that in the era of Napoleon, Wellington, James Madison, and Francis Scott Key, literacy was the exception, not the rule—and by a wide margin.

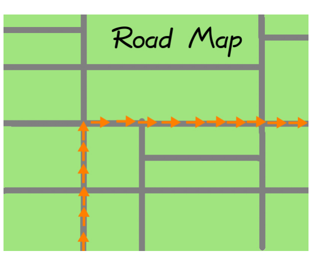
By analogy, an *innumerate* person is one who is not mathematically functional as pertains to those tasks which impact ordinary life, due to their mathematical ignorance. By "tasks which impact ordinary life" we mean someone incapable of using percentages, calculating compound interest, making basic business decisions, organizing personal finance with a budget, or reading a statistics-filled commercial advertisement.

By the way, did you notice that the words "illiterate" and "innumerate" were in italics? This is done to indicate that they are vocabulary terms. Any such terms will be in italics at the moment when they are defined, but in ordinary type whenever they appear subsequently.

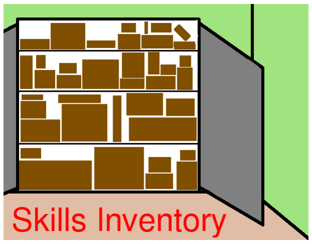
These boxes are about theory. For medium-skill and stronger students, seeing the theory is often quite helpful. It can explain where ideas come from, and these form hooks in your brain that connect the ideas together. Otherwise, memorizing a plethora of formulae is not very fun and is unlikely to be effective.

Alternatively, some students prefer to be told "just what to do" and "don't care where it comes from." While that attitude is depressing, I have also seen students with that attitude get an `A`.

Nonetheless, I request that every student read what they find in these boxes. If they do not understand it after the second reading, then they can move on without giving the theory box a third reading. This information is not essential, but everyone should try.

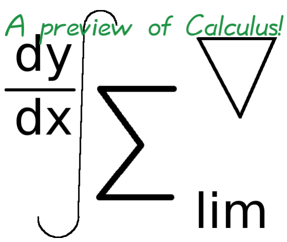


At the start of each module, you will find this box, and it lists the objectives of that module. I think it is extremely useful to know where the module is going, so that the reader knows what is important in the discussion. This is your road map to success, and it highlights the purposes of the module.



At the end of the module, this box appears. In it you will find an inventory of what was learned during the module. This is not decorative, but rather important. You must look at each bulleted item, and ensure that you have indeed acquired the skill listed. If not, then you should go back and learn more about the skills that you do not feel comfortable with.

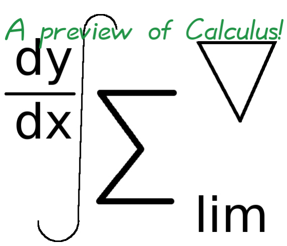
Also, I will highlight some vocabulary terms that were defined during the module. Both financial subjects and social-science subjects are vocabulary rich topics—much more so than most other subjects. Becoming an expert in the terminology is very important, not only for impressing people at job interviews for internships, co-ops, or volunteer positions, but also for understanding the instructor in higher-level courses.



Most students in business-related fields will take *Business Calculus* after taking *Finite Mathematics*. Therefore, since this book is "before calculus" for almost all students, you might be surprised to see calculus symbols floating in this box. I will occasionally make references to ideas to be found both in this book and in calculus, where presumably the calculus course will go more into depth. This is analogous to foreshadowing.

Foreshadowing is very important in literature, but for some reason, we don't often do it in mathematics. I'm not quite sure why. Certainly, until I really sat and thought about it, I don't think I did it very often at all in the classroom myself. My own teachers rarely did it, but I can remember one particularly excellent teacher, Prof. Richard Schwartz, then of the University of Maryland, now at Brown University, who did. He was teaching a course called *Abstract Algebra* that was toward the end of the Bachelor's Degree in math (or at the beginning of graduate study for math) and therefore those conceptual bridges were extremely important, and useful.

I hope that by highlighting the connections to calculus, I will build anticipation for that glorious subject—and at nearly all American universities, all science-related and business-related students must pass calculus.

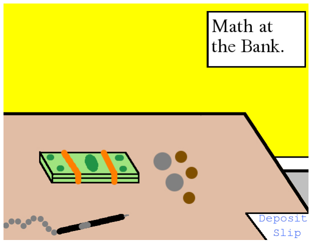


While the previous box is true for most students, the proliferation of advanced-placement courses, of many flavors, has changed the role of calculus in the lives of high school and university students. Therefore, it certainly does happen from time to time that students take *Finite Mathematics* after attempting some level of calculus.

Moreover, some high schools railroad their students into calculus to such an extent that all the topics in high school which do not prepare a student for that calculus-oriented senior year are considered to be very secondary. Therefore, topics like probability, permutations & combinations, matrices, and set theory, are sometimes left totally unexplored. Even material that is merely *somewhat* related to calculus—such as trigonometry or inequalities—is often underexplored. This is exacerbated by the fact that certain nation-wide examinations, such as the SAT, do not cover much mathematics other than algebra and geometry—but of course, those examinations cannot cover everything.

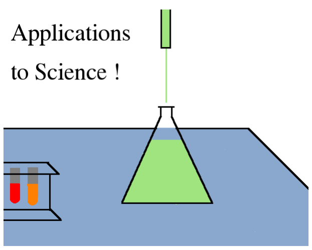
During this book, a student who has had calculus can enjoy all the useful and interesting bits of mathematics that were bypassed earlier in their education. Perhaps, they might even learn new uses for tools they are already familiar with.

For those students who do not need calculus, and have not yet had it, I hope I might inspire some of the more intrepid of them to explore the class—perhaps for real, or perhaps on a "pass/fail," "audit," or "pass/no credit" basis.



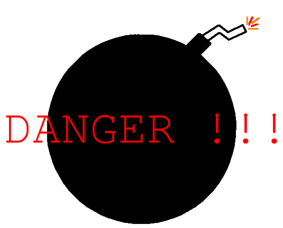
As I said earlier, it is extremely important to know how mathematics is used. Applications are outstandingly handy to help students understand mathematics. Some teachers are frustrated that they have to justify the importance of a topic to students, and they see applications as a sort of bribe or annoying proof of utility. I do not look at things that way myself. I find that applications allow students to relate the math to something they already understand.

Financial applications are doubly important. Not only do they help the student learn, but they also help keep the student out of bankruptcy and put the student on the road to success. The connections to finance, however, require a large vocabulary and also some grasp of the financial instruments that the math will serve. Therefore, these boxes highlight the technical knowledge from finance that students need—in order to understand the surrounding material. These boxes also contain a few very important life lessons. Instructors want their students to succeed, not just learn, and there are many pitfalls in the road to monetary success. In some ways, these are the most important boxes in the book, for they aim to prepare you to spot and hopefully avoid those pitfalls which could otherwise cause your financial ruin.



I have tried to avoid turning this book into a traditional science text. The focus is on business-related problems, with a few social-science related problems as well. Nonetheless, scientific applications are favorites of mine, especially of the variety where a mathematical concept can be used to solve problems very quickly, while the problems would otherwise be much harder. A particularly pleasant side-effect of math problems related to science is that they help us understand the world around us.

I have included many connections to science—and I probably should include more. I try to focus not only on the "big three" that most students have available in high school: biology, chemistry, and most dreaded of all—physics; I also focus on astronomy, ecology, archeology, meteorology, and geology—the little sciences which are sometimes but not always neglected.



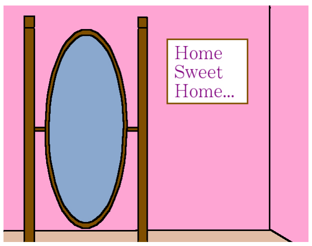
I am concerned that business-related students will ignore the science applications; and students in the social sciences will ignore financial applications. I'd like to address this.

- Having some small dose of scientific literacy is very important. Just a slight exposure to a few science topics can really improve one's world view. Often, people outside the sciences see these subjects as mysterious, or even frightening. Seeing how some scientific principles function can remove that veil of secrecy, and turn the foreign, strange and mysterious subjects into everyday topics.
- Having a tiny dose of scientific literacy can help a business-student land a position in the lucrative fields of bioengineering, biochemistry, pharmaceuticals or other medical-related businesses—or the computer hardware industry, which relies on chemistry more than you might imagine. My Bachelor's Degree and first Master's Degree were in Electrical & Computer Engineering, so accept my assurances on that.
- The other side is much easier to argue. Having a misunderstanding of financial facts can be fatal to a person's progress through life. There are financial errors that a person can make—and the repercussions will be felt for a decade or longer. Having a solid understanding of the mechanics of money is core to having a successful life.
- Regardless of your career track, and whatever jobs you might take during your lifetime, it is important that you learn how to manage your finances very well. This is particularly true about investing, where many intelligent people have made errors in the past.

I'm now going to share with you one of my favorite sayings:

"Money cannot buy happiness, but poverty purchases misery."
—Gregory V. Bard

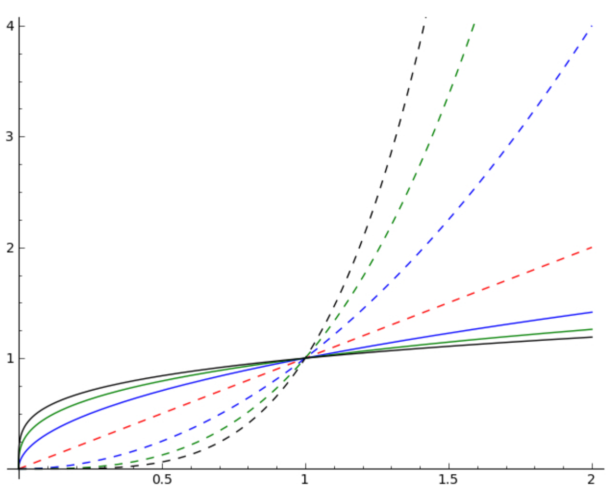
Some casual comments or passing statements are too small to be worthy of a box with an icon, and therefore will appear like this box and the previous box.



A Pause for Reflection. . .
When learning new concepts—especially large ones, or ones that are counter-intuitive, it is very good to stop, pause, and reflect on what you've learned. These breaks are great for the brain. They keep you from getting "study hypnosis," a phenomenon that can occur if you study for a very long continuous period. Here, I will invite you to think and reflect on some specific question related to the module, yet one that isn't quite mathematical. A few instructors might ask you to record your reflections in a homework journal, but perhaps very few. Mostly, this is just for your own comprehension, and to give your mind a momentary rest. This is not relaxation, instead it is reflection.

The computer algebra systems "Maple," "Matlab," "Mathematica," and "Magma" are very expensive tools that work like a highly sophisticated and capable graphing calculator—but running on a computer instead of a handheld device. The system SAGE is the free, open-source competitor to them all. Anybody with a web-browser can connect to the SAGE machines in Seattle, and perform computations on them. Also, because SAGE works through the web browser, the interface is remarkably simple to learn.

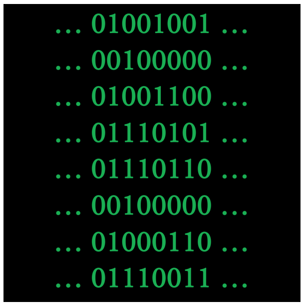
As it comes to pass, we will need Sage only occasionally, as a scientific calculator has sufficient power to do everything that we need done during this textbook.



Many functions are easiest explained by their graphs. In fact, we will explore several concepts through the power of graphs. The graphs in this book have been created by SAGE, you'll learn how to generate them yourself—just like a graphing calculator would, but with many additional and advanced features.

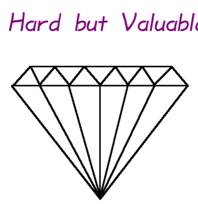
These boxes have no icon, but that space and a bit more is devoted to displaying the graph being discussed. The description of the graph and its underlying math will appear alongside. In this case, we're simultaneously graphing, on the interval $0 \le x \le 2$, the following set of functions:

$$\left\{ x^4,\ x^3,\ x^2,\ x,\ x^{1/2},\ x^{1/3},\ x^{1/4} \right\}$$



These boxes indicate a connection with computer science. They occur primarily toward the end of the book. That's because certain topics—such as combinatorics, probability, logic, and binary—are vital to how computers operate inside.

The connections are often surprising and fascinating. It is good to learn about these things, so that computers do not seem like "magic." Understanding how computers work inside will make you a better user of technology.



Of course, we all know that diamonds are valuable, but they are also quite hard and have many industrial uses. This type of box, marked with a diamond, represents material that is both hard and valuable, just like diamonds are. Some instructors will include this advanced material, to challenge you and take you to a new level of mastery of the material. Other instructors will bypass this material, if they feel it is too difficult, or if they feel it will bring them off their lecture schedule.

I sincerely hope that you will attempt to tackle the harder material even if your instructor doesn't. However, you could also check with your instructor to see if that harder material is mandatory or not.

This book is a work in progress, and is not quite finished yet. There will be boxes like this, showing you were a piece would go, if it were ready.

Actually, at this point of September 2014, there are only a few spots that are not yet written. Therefore, you won't see this box very often at all.

At this point, I have now covered all of the types of boxes of this book, and that was the primary purpose of this preface. I just have a few more brief but useful things to tell you about the textbook very quickly, and then you'll be ready to dive in to the body of the textbook.

In this book, you'll be doing a lot of calculations. Some will be simple, and some will be complicated. You'll almost always use a calculator for these math problems. As you might have noticed, calculators usually display between 8 to 11 digits, but few situations really call for that many digits of accuracy.

The policy in this book is that we will be using 6 digits of accuracy for numbers, except in rare situations when we use 9 digits, and those will be very clearly marked. Furthermore, I will not round, but instead, I will slice off the numbers after the sixth digit. That means when your calculator tells you

$$\sqrt{5} = 2.23606797749979\cdots$$

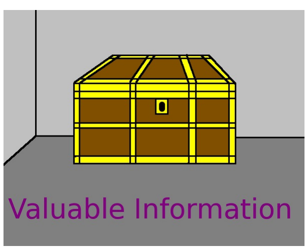
then I will be writing $2.23606\cdots$ in the book and not $2.23607\cdots$. Don't worry about why, however, because we'll be discussing that in detail fairly soon.

However, if your answer (for any problem in this book) matches for all digits but the last digit, and doesn't match on the last digit, then that's totally okay. It is standard practice that the last digit written is not as reliable as the ones that come before it.

We will learn all about significant digits, rounding, and scientific notation soon enough, in our first module. That's Module 0.1: "Scientific Notation and Significant Digits."

Most readers have no reason to be worried about, or be interested in, rounding error. However, if you are deeply curious, the rounding/truncation policy of this book is as follows:

- Numbers that are part of "shown work," demonstrating how to solve problems in examples and so forth, will have six significant figures shown.
- Final answers that are dollar-values, unless enormous, should be truncated to the penny. (e.g. $ 5.10, not $ 5.1, nor $ 5.102.) Very large dollar amounts will be truncated to the nearest dollar. (e.g. $ 1,234,567 and not $ 1,234,567.89.)
- Final answers that are percentages should be truncated to the basis point—that means to 1% of 1%. (e.g. 7.12%, not 7.1%, nor 7.123%.)
- Answers that are integers should be written as integers. (e.g. 6, not 6.0, nor 6.00.)
- Other final answers should be truncated to six significant figures.

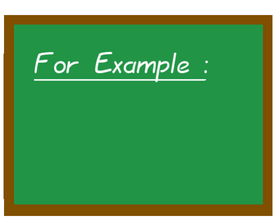


Another feature of this book is "Chapter 0," which is designed to measure your background.

Everyone forgets a few things over summer break and winter break, and therefore it is unsurprising if you have forgotten a small algebraic technique or two. There are two diagnostics in Chapter 0 which are designed to measure you. Diagnostic One measures your skill with a calculator—everyone gets a bit rusty—and Diagnostic Two measures your algebra skills.

The diagnostic is just a sequence of questions for you to solve. At the end of the diagnostic, the answers are given. Based on which questions you might have gotten wrong, you'll be directed to read this or that module for review. By this mechanism, you'll only review what you've forgotten, and no one will waste your time asking you to review things that you already know.

Please be certain to do both diagnostics, as they will do you no benefit if you leave them unattempted.



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It is necessary to have a word about the notation used in algebra. Let's say that in middle school, when you first learned about algebra, that you were asked to perhaps solve the following problem:

$$17x + 25 = 12x + 55$$

It is possible that you might have written something in the following style:

$$\begin{array}{rcl}
17x + 25 & = & 12x + 55 \\
-25 & & -25 \\
\hline
17x & = & 12x + 30 \\
-12x & & -12x \\
\hline
5x & = & 30 \\
\div 5 & & \div 5 \\
\hline
x & = & 6
\end{array}$$

However, you will not see that style of notation in this book. Instead, the high-school style notation will be used, as presented in the next box.

Continuing with the previous box, the high-school style notation of solving $17x + 25 = 12x + 55$ would look something like the following:

$$\begin{aligned} 17x + 25 &= 12x + 55 \\ 17x &= 12x + 30 \\ 5x &= 30 \\ x &= 6 \end{aligned}$$

As you can see, it is completely transparent as to what the thought process is, and so this notation conveys 100% of the information about how to solve the problem. However, it takes up considerably less space than the notation of the previous box. Therefore, that's how we'll be seeing algebra done in this textbook. I hope you don't mind, but this is unavoidable. Otherwise the textbook would be twice as large as it already is, and it already has 1396 pages.

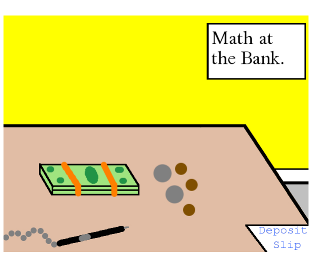
Nonetheless, I'm sure you'll find that I'm showing much more detail in the calculations than the typical university-level textbook shows.

If you feel as though you need the additional symbols and steps of the middle-school style notation, then you probably do not have enough skill in algebra to be able to read this textbook at all.

You should talk to your instructor about switching to a different mathematics course, such as perhaps *Intermediate Algebra*.

Algebra is the key to this textbook. Without skill in algebra, there is no hope of understanding what we are talking about. Diagnostic Two, in Chapter Zero, will help you determine whether you have sufficient knowledge of algebra.

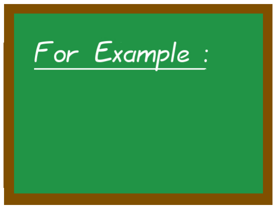
I'd like to share with you a cool problem that I call the 654,321-example. It begins in the next box, and I hope that you'll keep it in mind throughout the entire course.



Imagine you are a mid-level manager, and you're supervising a small division whose operating costs are \$ 654,321 per year. You have an identical twin who has an identical small division whose operating costs are the same, except that your twin has not taken this course.

With all the mathematics that you learn during this semester, including cost-benefit analysis, break-even analysis, annual equivalent rates, sinking funds, commercial paper, systems of inequalities, quadratic optimization, forecasting with regressions, portfolio balancing, probability, expected value, amortization, and so forth—let us imagine that you find five opportunities to adjust your division's practices to cut costs. That's a bit pessimistic because I'd hope that you find more than five, but to be even further pessimistic, let's suppose that each change only saves you 2%. Again, I hope you'd save more than 2%, but I'm trying to be conservative in my estimates.

We will continue the analysis in the next box.



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Continuing with the analysis of the previous box, now we're going to see what happens if your division (and that of your identical twin) each earns revenue equal to $ 620,000. What will the profit (in dollars) be for the year?

We'll do this in two stages. First, we'll figure out your costs.

	Your Division	Your Twin's Division
Starting Costs	654,321.00	654,321.00
After 1st Change	641,234.58	654,321.00
After 2nd Change	628,409.88	654,321.00
After 3rd Change	615,841.69	654,321.00
After 4th Change	603,524.85	654,321.00
After 5th Change	591,454.35	654,321.00

As you can see, your twin wasn't able to make the changes that you made, since your twin did not take this course.

Take a moment to verify that each change in the previous box indeed cut costs by 2%. For example, between the fourth and fifth change,

$$(603,524.85)(0.98) = 591,454.35$$

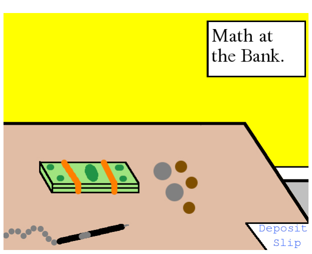
as expected. Verify for yourself another one of the lines of that table.

The problem is continued in the next box.

Continuing from the previous box, now that we've computed the costs for your division and your twin's division, let's see what happens if both divisions earn $ 620,000 in revenue, just as an example.

	Your Division		Your Twin's Division	
Revenue:	$ 620,000.00		$ 620,000.00	
Actual Costs:	$ -591,454.35		$ -654,321.00	
Profit/Loss:	$ +28,545.65	profit!	$ -34,321.00	loss!
Outcome:	You are promoted.		The twin gets fired.	

Given that the divisions are basically the same and earned the same revenue, surely it is shocking that one division had a profit of $ 28,545 while the other had a loss of $ 34,321. I'm certain we can all agree that higher management is entirely justified in firing your twin, and putting his division under your capable management.



Of course, I hope you'll find more than five opportunities, and I sincerely hope they'll garner you more than a 2% reduction in costs, each. Nonetheless, as you can see, even these modest cost-controls are capable of turning a $ 34,321 loss into a $ 28,545 profit, and that can easily be the difference between being promoted and getting sacked.

So remember, what we study here will be very concrete and applicable to the real world. If you pay close attention, then you can gain a competitive advantage in the workplace after you graduate.

This ends our coverage of "How to Use this Book." I sincerely hope that you will enjoy exploring the chapters which follow, rich with examples from the real world.