On The Distributive Law

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The distributive law is a powerful tool in mathematics. However, it is important to know when to use it, and when not to use it.

Some Things are Distributive

- $x(y+z) = xy + xz$, allowing maneuvers such as $2(y+z) = 2y + 2z$, or $-(y+z) = -y - z$.
- $\frac{(y+z)}{x} = \frac{y}{x} + \frac{z}{x}$, allowing maneuvers such as $\frac{(y+z)}{3} = \frac{y}{3} + \frac{z}{3}$.

Others Have Alternative Formulae

- $(x+y)^2 \neq x^2 + y^2$ but rather $(x+y)^2 = x^2 + 2xy + y^2$.
- $(x+y)^3 \neq x^3 + y^3$ but rather $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.
- $(x+y)^4 \neq x^4 + y^4$ but rather $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.
- $(x+y)^{-1} \neq x^{-1} + y^{-1}$ but rather $(x+y)/xy$.
- $\sin(x+y) \neq (\sin x) + (\sin y)$ but rather $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$.
- $\cos(x+y) \neq (\cos x) + (\cos y)$ but rather $\cos(x+y) = \cos(x)\cos(y) + \sin(x)\sin(y)$.
- $e^{x+y} \neq e^x + e^y$ but rather $e^{x+y} = e^x e^y$.
- Most interestingly, $\tan(x+y) \neq (\tan x) + (\tan y)$ but rather

$$\tan(x+y) = \frac{\sin(x)\cos(y) + \cos(x)\sin(y)}{\cos(x)\cos(y) + \sin(x)\sin(y)} = \frac{\cot(y) + \cot(x)}{\cot(x)\cot(y) + 1} = \frac{\tan(x) + \tan(y)}{\tan(y)\tan(x) + 1}$$

Others Have No Easy Form

- $\log(x+y) \neq \log(x) + \log(y)$ but rather $\log(xy) = (\log x) + (\log y)$.
- $\ln(x+y) \neq \ln(x) + \ln(y)$ but rather $\ln(xy) = (\ln x) + (\ln y)$.
- $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$ but rather $\sqrt{xy} = (\sqrt{x})(\sqrt{y})$.
- $\sqrt[3]{x+y} \neq \sqrt[3]{x} + \sqrt[3]{y}$ but rather $\sqrt[3]{xy} = (\sqrt[3]{x})(\sqrt[3]{y})$.
- $(ab)! \neq (a!)(b!)$ and has no easy form.

The statements on this page are intended to apply to all x, y, z in the real numbers. For example, obviously $(x+y)^4 = x^4 + y^4$ if either x or y are zero.